Introduction to the Logic of Conditionals

Paul Egré and Mikaël Cozic

ESSLLI 2008
20th European Summer School in Logic, Language and Information
4–15 August 2008
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Introduction to the Logic of Conditionals

Course Material. 20th European Summer School in Logic, Language and Information (ESSLLI 2008), Freie und Hansestadt Hamburg, Germany, 4–15 August 2008

Introduction to the logic of conditionals

ESSLLI 2008
Week 1 - August 4-8

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Description of the course

Welcome to the ESSLLI 2008 course "Introduction to the logic of conditionals." This course is *foundational*, which means that our aim is to provide an accessible introduction to the logic of conditionals, suitable for students coming from different disciplines, whether logic, natural language semantics, computer science, or philosophy. Our ambition is to provide you with the basic tools that have become standard in any discussion of conditionals in natural language, in particular in the areas of philosophical logic and natural language semantics. More than that, our goal is is to lead you as efficiently as possible to the aspects of the study of conditionals that are particularly active today and could become an object of further research for you.

The course does not presuppose prior knowledge of conditional logics. The only background we assume is some knowledge of classical logic, namely propositional and first-order logic. Some previous knowledge of modal logic will help, but is not required. Because the starting point of any analysis of conditionals is also the simplest, however, namely the truth-functional analysis in terms of material conditional, even those who would have had little exposure to logic (as opposed to linguistics, in particular) are welcome to attend the class.

In the present document, we only provide a day-by-day description of the course and a list of suggested readings. The slides of the course will be made available online by the time of the course, at the following address :

http://paulegre.free.fr/Teaching/ESSLLI_2008/index.htm

Initially, our goal was to provide a comprehensive reader, containing all the papers that are on our reading list. Because of copyright issues, however, and for the sake of efficiency, we decided to only link the papers, whenever possible. Some additional papers, which are particularly hard to find even online, will be made available to participants of the class upon request (at the time of the conference).

Our hope is that you will enjoy the course and find it useful. We are working on it !

Mikaël Cozic and Paul Égré

The course day by day

Remember that the list of suggested readings is only suggestive : it means that you are not supposed to have read all the indicated papers in advance. Rather, the course will prime you on aspects of the papers that you can focus on more comfortably and more efficiently in attending the class.[1]

Monday, August 4 : The Stalnaker-Lewis analysis of conditionals

• Review of the strengthes and inadequacies of the material conditional analysis of NL conditionals. Stalnaker's analysis in terms of selection functions. The limit and unicity assumptions (Lewis). Intermediate systems in terms of correspondence functions. Adjudicating between Stalnaker and Lewis's systems. Problems for both analyses. Recent generalizations : Girard's analysis, Schlenker's analysis.

Basic reading	• Robert Stalnaker 1968, "A Theory of Conditionals", in W. Harper, R. Stalnaker and G. Pearce (eds), *Ifs*, pp. 41-55. (available from the instructors, or on Google Scholar).
Further reading	• David Lewis (1973), "Counterfactuals and Comparative Similarity", *Journal of Philosophical Logic* 2 :4, pp. 418-446, http://www.springerlink.com/content/f3536272w2771x33/
Recent perspectives	• P. Schlenker (2003), "Conditionals as definite descriptions" http://www.linguistics.ucla.edu/people/schlenker/Conditionals.pdf • P. Girard (2006), "From Onions to Broccoli : Generalizing Lewis's Counterfactual Logic" http://www.stanford.edu/~pgirard/jancl-paper.pdf

Tuesday, August 5 : Conditionals as restrictors

• Are conditionals binary connectives ? The Lewis-Kratzer analysis of conditionals as adverbials restrictors. Kratzer's "doubly-relative" analysis of modals. Interaction between quantifiers and conditionals. Gibbard's riverboat example. Iatridou and von Fintel's counterexamples.

Basic reading	• David Lewis 1975, "Adverbs of Quantification", repr. in D. Lewis, *Papers in Philosophical Logic*, Cambridge UP. • Angelika Kratzer 1991, "Conditionals", in A. von Stechow and D. Wunderlich (eds.), *Semantics : an International Handbook of Contemporary Research*, pp. 639-650. (available from the instructors).
Further reading	• Allan Gibbard 1980, "Two Theories of Conditionals", in W. Harper, R. Stalnaker and G. Pearce (eds), *Ifs* (available from the instructors).
Recent perspectives	• Kai von Fintel & Sabine Iatridou (2002), "If and When If-Clauses can restrict Quantifiers" http://web.mit.edu/fintel/www/lpw.mich.pdf

[1] Disclaimer and warning : all the links we provide to papers are *open* links to material available from the internet. Some of these links may only be functional if your home institution has a subscription to the journal.

Wednesday, August 6 : Conditionals and Rational Belief Change

• Probabilistic and set-theoretic views of rational belief change. The AGM framework of belief dynamics. The Ramsey test. Probability of conditionals and conditional probability. Adams's Thesis. Adams's probabilistic logic and its relationship with Stalnaker's semantics. Assertion and conditionals.

Basic Reading	• E. Adams (1998), *A Primer of Probability Logic* [chapters 6 and 7], CSLI Publications, Stanford.
Further Reading	• Jackson, F. (1979) "On Assertion and Indicative Conditionals", *The Philosophical Review*, vol.88, n°4, pp. 565-89 (available from JSTOR).
Recent perspectives	• S. Kaufmann (2004), "Conditioning against the Grain : Abduction and Indicative Conditionals", *Journal of Philosophical Logic* 33 :583-606, http://ling.northwestern.edu/~kaufmann/Offprints/JPL_2004_Grain.pdf

Thursday, August 7 : Triviality Results and their implications

• Lewis's Triviality Results. Gärdenfors's qualitative version of the triviality results. Conditionalization vs. imaging. Three responses to triviality results : (i) the No-Truth Value conception (Edgington), (ii) contextualist semantics (Bradley), (iii) refinement of probabilistic logic (McGee). Connections with conditionals as modality restrictors.

Basic Reading	• D. Lewis (1976), "Probability of Conditionals and Conditional Probability", *The Philosophical Review*, Vol. 85, No. 3. (Jul., 1976), pp. 297-315.(available from JSTOR).
Further Reading	• D. Edgington (1995), "On Conditionals", *Mind*, vol. 104, n°414, 1995, pp. 235-329.(available from JSTOR). • V. McGee (1989), "Conditional Probabilities and Compounds of Conditionals", *The Philosophical Review*, vol.98, No.4., pp. 485-541.(available from JSTOR).
Recent Perspectives	• R. Bradley (2002), "Indicative Conditionals", *Erkenntnis* 56 : 345-378, 2002, http://www.springerlink.com/content/n4qq7nm3xg5llcxy/fulltext.pdf.

Friday, August 8 : Counterfactual Conditionals

• Dualist vs. unified theories of indicative and subjunctive conditionals. Counterfactuality as implicature or as presupposition. Stalnaker's pragmatic constraint. Tense and mood in counterfactuals. Dynamic semantics for counterfactuals (Veltman).

Basic Reading	• R. Stalnaker (1975), "Indicative Conditionals", *Philosophia 5*, repr. in R. Stalnaker *Context and Content*, Oxford 1999, http://www.springerlink.com/content/u543308t7871g193/fulltext.pdf.
Further Reading	• K. von Fintel (1997), "The Presupposition of Subjunctive Conditionals", *MIT Working Papers in Linguistics*, O. Percus & U. Sauerland (eds.), mit.edu/fintel/www/subjunctive.pdf
Recent perspectives	• S. Iatridou (2000), "The Grammatical Ingredients of Counterfactuality", *Linguistic Inquiry*, vol. 31, 2, 231-270. • F. Veltman (2005), "Making Counterfactual Assumptions", *Journal of Semantics* 22 : 159-180, http://staff.science.uva.nl/~veltman/papers/FVeltman-mca.pdf

Note : For reasons of time and coherence, we decided not to include material on so-called relevance or "biscuit" conditionals (conditionals of the form "if you are hungry, there are biscuits in the kitchen"). We will be happy to provide reference about those, but may not have the time to talk about them in great detail.

Introduction to the Logic of Conditionals

ESSLLI 2008

M. Cozic & P. Egré

IHPST/Paris 1, CNRS, DEC-ENS
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What are conditional sentences?

If P then Q

(1) If it's a square, then it's rectangle.

(2) If you strike the match, it will light.

(3) If you had struck the match, it would have lit.

Role of conditionals in mathematical, practical and causal reasoning.

Antecedent and consequent

(4) If P then Q

P: antecedent, protasis
Q: consequent, apodosis

Conditionals without "if...then..."

- **Imperative** (Bhatt and Pancheva 2005)

(5) a. Kiss my dog and you'll get fleas.
b. If p, q.

(6) a. Kiss my dog or you'll get fleas.
b. If $\neg p$, q.

- **No...No...** (Lewis 1972)

(7) a. No Hitler, no A-bomb
b. If there had been no Hitler, there would have been no A-bomb.

- **Unless**

(8) a. Unless you talk to Vito, you will be in trouble.
b. If you don't talk to Vito, you will be in trouble.

How to analyze conditional sentences?

Main options we shall discuss in this course:

- Conditionals as truth-functional binary connectives: material conditional
- Conditionals as non-truth-functional, but truth-conditional binary connectives: Stalnaker-Lewis
- Conditionals as truth-conditional quantifier restrictors ($\neq$ binary connectives): Kratzer
- Conditionals as non-truth-conditional binary connectives: Edgington,...

Indicative vs. Subjunctive conditionals

- Another issue:

  (9) If Oswald did not kill Kennedy, someone else did.

  (10) If Oswald had not killed Kennedy, someone else would have.

- See Lecture 5

Roadmap

1. Lecture 1: Stalnaker-Lewis semantics
2. Lecture 2: Conditionals as restrictors
3. Lecture 3: Conditionals and rational belief change
4. Lecture 4: Triviality results and their implications
5. Lecture 5: indicative vs subjunctive

Where to look for Stalnaker (1968), Gibbard (1980), Kratzer (1991):

http://paulegre.free.fr/Teaching/ESSLLI_2008/stalnaker.pdf

http://paulegre.free.fr/Teaching/ESSLLI_2008/gibbard.pdf

http://paulegre.free.fr/Teaching/ESSLLI_2008/Kratzer1991.pdf

1. The Stalnaker-Lewis analysis of conditionals

The Material Conditional

The material conditional

- Sextus Empiricus, *Adv. Math., VIII*: Philo used to say that the conditional is true when it does not start with the true to end with the false; therefore, there are for this conditional three ways of being true, and one of being false
- Frege to Husserl 1906: Let us suppose that the letters 'A' and 'B' denote proper propositions. Then there are not only cases in which A is true and cases in which A is false; but either A is true, or A is false; *tertium non datur*. The same holds of B. We therefore have four combinations:

  A is true and B is true
  A is true and B is false
  A is false and B is true
  A is false and B is false

  Of those the first, third and fourth are compatible with the proposition “if A then B", but not the second.

The truth-functional analysis

$[\![\phi]\!]$	$[\![\psi]\!]$	$[\![(\phi \rightarrow \psi)]\!]$
1	1	1
1	0	0
0	1	1
0	0	1

- $[\![\phi \rightarrow \psi]\!] = 0$ iff $[\![\phi]\!] = 1$ and $[\![\psi]\!] = 0$
- $[\![\rightarrow]\!] = cond : \{0,1\} \times \{0,1\} \rightarrow \{0,1\}$
  $cond(x, y) = 0$ iff $x = 1$ and $y = 0$
- $[\![\phi \rightarrow \psi]\!] = [\![\neg(\phi \wedge \neg\psi)]\!]$

Binary Boolean functions

		f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}	f_{16}
1	1	0	0	0	0	1	0	0	0	1	1	1	0	1	1	1	1
1	0	0	0	0	1	0	0	1	1	0	0	1	1	0	1	1	1
0	1	0	0	1	0	0	1	0	1	0	1	0	1	1	0	1	1
0	0	0	1	0	0	0	1	1	0	1	0	0	1	1	1	0	1
p	q	$\bot$	$\nmid$	$\nleftarrow$	$\nrightarrow$	$\wedge$	$\neg p$	$\neg q$	$\leftrightarrow$	$\nleftrightarrow$	q	p	$\mid$	$\rightarrow$	$\leftarrow$	$\vee$	$\top$

- Assuming a two-valued logic, and the conditional to be a binary connective: no other boolean function is a better candidate to capture the conditional's truth-conditions
- At least: the material conditional captures the falsity conditions of the indicative conditional of natural language.

Propositional validity

- ϕ is a tautology or logical truth iff $[\![\phi]\!]$=1 for all assignment of truth-value to the propositional atoms of ϕ. ($\models \phi$)
- ϕ is a logical consequence of a set Γ of formulae iff every assignment of truth-value that makes all the formulae of Γ true makes ϕ true. ($\Gamma \models \phi$)

“Good" validities

- $\phi \rightarrow \psi, \phi \models \psi$ (modus ponens)
- $\phi \rightarrow \psi, \neg\psi \models \neg\phi$ (modus tollens)
- $(\phi \vee \psi) \models \neg\phi \rightarrow \psi$ (Stalnaker's “direct argument"; aka disjunctive syllogism)
- $\models (((\phi \wedge \psi) \rightarrow \chi) \leftrightarrow (\phi \rightarrow (\psi \rightarrow \chi)))$ (import-export)
- $\models [(\phi \vee \psi) \rightarrow \chi] \leftrightarrow [(\phi \rightarrow \chi) \wedge (\psi \rightarrow \chi)]$ (simplification of disjunctive antecedents)

"Bad" validities

- $\neg\phi \models (\phi \rightarrow \psi)$ (falsity of the antecedent)
- $\phi \models (\psi \rightarrow \phi))$ (truth of the consequent)
- $(\phi \rightarrow \psi) \models (\neg\psi \rightarrow \neg\phi))$ (contraposition)
- $(\phi \rightarrow \psi), (\psi \rightarrow \chi) \models (\phi \rightarrow \chi))$ (transitivity)
- $(\phi \rightarrow \psi) \models ((\phi \wedge \chi) \rightarrow \psi)$ (antecedent strengthening)
- $\models \neg(\phi \rightarrow \psi) \leftrightarrow (\phi \wedge \neg\psi)$ (negation)

Why "bad" validities?

Undesirable validities w.r.t. natural language and ordinary reasoning:

- "Paradoxes of material implications" (Lewis). The paradox of the truth of the antecedent:

(11) a. John will teach his class at 10am.
b. ??Therefore, if John dies at 9am, John will teach his class at 10am.

(12) a. John missed the only train to Paris this morning and had to stay in London.
b. ??So, if John was in Paris this morning, John missed the only train to Paris this morning and had to stay in London.

Contraposition, Strengthening, Transitivity

(13) a. If Goethe had lived past 1832, he would not be alive today.
b. ??If Goethe was alive today, he would not have lived past 1832.

(14) a. If John adds sugar in his coffee, he will find it better.
b. ??If John adds sugar and salt in his coffee, he will find it better.

(15) a. If I quit my job, I won't be able to afford my apartment. If I win a million, I will quit my job.
b. ??If I win a million, I won't be able to afford my apartment. (Kaufmann 2005)

Qualms about non-monotonicity

- Does order matter? (von Fintel 2001)

(16) If I win a million, I will quit my job. ??If I quit my job, I won't be able to afford my apartment.

(17) If the US got rid of its nuclear weapons, there would be war. But if the US and all nuclear powers got rid of their weapons, there would be peace.

(18) If the US and all nuclear powers got rid of their nuclear weapons there would be peace; ?? but if the US got rid of its nuclear weapons, there would be war.

- Non-monotony seems less consistent when conjuncts are reversed.

Negation of a conditional

(19) a. It is not true that if God exists, criminals will go to heaven.
b. (??) Hence God exists, and criminals won't go to heaven.

The expected understanding of negation is rather:

(20) If God exists, criminals won't go to heaven.

(21) $\neg$(if p then q) = if p then $\neg q$

Several diagnoses

- The examples raise a problem for the pragmatics of conditionals, and do not call for a revision of the semantics. (Quine 1950 on indicative conditionals, Grice 1968, Lewis 1973).
- The examples call for a revision of the semantics of conditionals (Quine 1950 on counterfactual conditionals, Stalnaker 1968, Lewis 1973)

Limits of truth-functionality

Whatever the proper analysis of the contrafactual conditional may be, we may be sure in advance that it cannot be truth-functional; for, obviously ordinary usage demands that some contrafactual conditionals with false antecedents and false consequents be true and that other contrafactual conditionals with false antecedents and false consequents be false (Quine 1950)

(22) If I weighed more than 150 kg, I would weigh more than 100 kg.

(23) If I weighed more than 150 kg, I would weigh less than 25 kg.

Suppose I weigh 70 kg. Then the antecedent and consequent of both conditionals are presently false (put in present tense), yet the first is true, the second false.

Strict conditionals

Motivation: take "if P then Q" to mean "necessarily, if P then Q" (C.I. Lewis)

- $\phi := \ p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \rightarrow \phi \mid \Box\phi$
- Abbreviation: $\phi \hookrightarrow \psi := \Box(\phi \rightarrow \psi)$
- Semantics: Kripke model $M = \langle W, R, I \rangle$.
  - (i) $M, w \models p$ iff $w \in I(p)$
  - (ii) $M, w \models \neg\phi$ iff $M, w \not\models \phi$
  - (iii) $M, w \models (\phi \wedge \psi)$ iff $M, w \models \phi$ and $M, w \models \psi$
    $M, w \models (\phi \vee \psi)$ iff $M, w \models \phi$ or $M, w \models \psi$
    $M, w \models (\phi \rightarrow \psi)$ iff $M, w \not\models \phi$ or $M, w \models \psi$
  - (iv) $M, w \models \Box\phi$ iff for all v s.t. vRw, $M, v \models \phi$
- Validity: $\models \phi$ iff for every M and every w in M, $M, w \models \phi$.

Consequences

- The strict conditional “solves" the paradoxes of material implication. In particular: $\not\models (p \hookrightarrow (q \hookrightarrow p)$. Why? Construct model for $\Diamond(p \wedge \Diamond(q \wedge \neg p))$.
- However, the strict conditional is still monotonic:

(24) $\Box(p \rightarrow q) \models \Box(\neg q \rightarrow \neg p)$

(25) $\Box(p \rightarrow q) \models \Box(p \wedge r \rightarrow q)$

(26) $\Box(p \rightarrow q), \Box(q \rightarrow r) \models \Box(p \rightarrow r)$

Conclusion: must do better.

Stalnaker's logic

Stalnaker's analysis: background

How do we evaluate a conditional statement?

- *First, add the antecedent hypothetically to your stock of beliefs; second, make whatever adjustments are required to maintain consistency (without modifying the hypothetical belief in the antecedent; finally, consider whether or not the consequent is then true. (Stalnaker 1968)*
- *Consider a possible world in which A is true, and which otherwise differs minimally from the actual world. "If A then B" is true (false) just in case B is true (false) in that possible world. (Stalnaker 1968)*

Stalnaker's logic

- $\phi := \ p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \rightarrow \phi \mid \phi > \phi$
- Stalnaker-Thomason model: $M = \langle W, R, I, f, \lambda \rangle$, where $\langle W, R, I \rangle$ is reflexive Kripke model, λ absurd world (inaccessible from and with no access to any world), and $f : \wp(W) \times W \rightarrow W$ is a selection function satisfying:

(cl1) $f(\llbracket \phi \rrbracket, w) \in \llbracket \phi \rrbracket$

(cl2) $f(\llbracket \phi \rrbracket, w) = \lambda$ only if there is no w' s.t. wRw' and $w' \in \llbracket \phi \rrbracket$

(cl3) if $w \in \llbracket \phi \rrbracket$, then $f(\llbracket \phi \rrbracket, w) = w$

(cl4) if $f(\llbracket \phi_2 \rrbracket, w) \in \llbracket \phi_1 \rrbracket$ and $f(\llbracket \phi_1 \rrbracket, w) \in \llbracket \phi_2 \rrbracket$, then $f(\llbracket \phi_2 \rrbracket, w) = f(\llbracket \phi_1 \rrbracket, w)$

(cl5*) if $f(\llbracket \phi \rrbracket, w) \neq \lambda$, then $f(\llbracket \phi \rrbracket, w) \in R(w)$

Semantics

(i) $M, w \models p$ iff $w \in I(p)$
(ii) $M, w \models \neg\phi$ iff $M, w \not\models \phi$
(iii) $M, w \models (\phi \wedge \psi)$ iff $M, w \models \phi$ and $M, w \models \psi$
$M, w \models (\phi \vee \psi)$ iff $M, w \models \phi$ or $M, w \models \psi$
$M, w \models (\phi \rightarrow \psi)$ iff $M, w \not\models \phi$ or $M, w \models \psi$
(iv) $M, w \models (\phi > \psi)$ iff $M, f(\llbracket \phi \rrbracket, w) \models \psi$

- For every formula ϕ: $M, \lambda \models \phi$.

Looking at the clauses

- cl1 ensures that $\phi > \phi$, cl3 that no adjustment is necessary when the antecedent already holds at a world.
- cl2 and cl5*: selected world is absurd when antecedent is impossible.
- cl4: coherence on the ordering induced by the selection function.

Axiomatics

Stalnaker's C2

$\Box\phi =_{df} (\neg\phi > \phi)$
$\Diamond\phi =_{df} \neg(\phi > \neg\phi)$
$(\phi <> \psi) =_{df} ((\phi > \psi) \wedge (\psi > \phi))$
(PROP) All tautological validities
(K) $(\Box\phi \wedge \Box(\phi \rightarrow \psi)) \rightarrow \Box\psi$
(MP) From ϕ and $(\phi \rightarrow \psi)$ infer ψ
(RN) From ϕ infer $\Box\phi$
(a3) $\Box(\phi \rightarrow \psi) \rightarrow (\phi > \psi)$
(a4) $\Diamond\phi \rightarrow ((\phi > \psi) \rightarrow \neg(\phi > \neg\psi))$
(a5) $(\phi > (\psi \vee \chi)) \rightarrow ((\phi > \psi) \vee (\phi > \chi))$
(a6) $((\phi > \psi) \rightarrow (\phi \rightarrow \psi))$
(a7) $((\phi <> \psi) \rightarrow ((\phi > \chi) \rightarrow (\psi > \chi))$

Important consequence

- $\models (\phi \hookrightarrow \psi) \rightarrow (\phi > \psi) \rightarrow (\phi \rightarrow \psi)$
- Stalnaker's conditional is intermediate between the strict and the material conditional (a "variably strict conditional", in Lewis's terms).

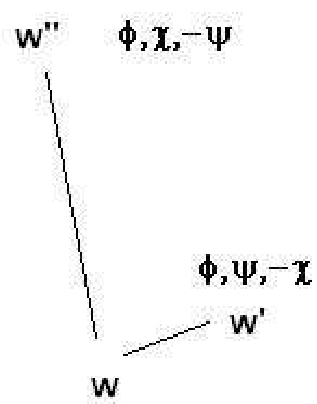
Invalidities

None of the “bad" validities comes out valid in Stalnaker's logic

- (FA) $\neg\phi \nvDash (\phi > \psi)$
- (TC) $\phi \nvDash (\psi > \phi))$
- (C) $(\phi > \psi) \nvDash (\neg\psi > \neg\phi)$
- (S) $(\phi > \psi) \nvDash ((\phi \wedge \chi) > \psi)$
- (T) $(\phi > \psi), (\psi > \chi) \nvDash (\phi > \chi)$

Example: monotonicity failure

- $(\phi > \psi) \nvDash ((\phi \wedge \chi) > \psi)$.
- Take $w' = f(\llbracket \phi \rrbracket, w)$, such that $w' \models \psi$, and $w'' = f(\llbracket \phi \wedge \chi \rrbracket, w)$, such that $w'' \nvDash \psi$.



Weak monotonicity

Monotonicity is lost, but a weakened form is preserved:

(CV) $(((\phi > \psi) \wedge \neg(\phi > \neg\chi)) \rightarrow ((\phi \wedge \chi) > \psi)))$

Positive properties

- Negation: $\Diamond\phi \models \neg(\phi > \psi) \leftrightarrow (\phi > \neg\psi)$
- Conditional excluded middle: $(\phi > \psi) \vee (\phi > \neg\psi)$
- Modus ponens: $\phi, (\phi > \psi) \models \psi$

Lewis's logic

Lewis's objections

D. Lewis objects to two aspects of Stalnaker's system:

- Uniqueness assumption: for every world w, there is at most one closest ϕ-world to w.
- Limit assumption: for every world w, there is at least one closest ϕ-world to w.

The Uniqueness Assumption

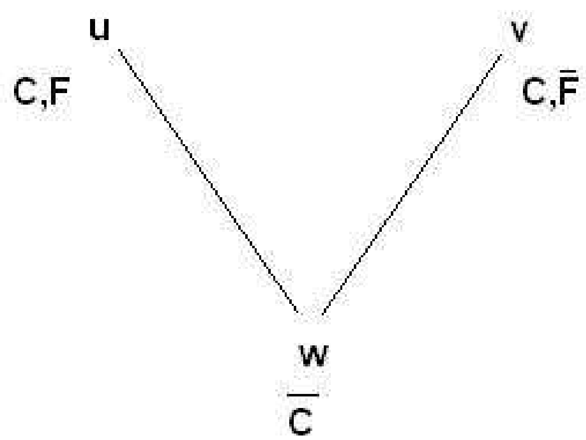
Conditional excluded middle

(27) (CEM) $(\phi > \psi) \vee (\psi > \neg\psi)$

(28) a. If Bizet and Verdi were compatriots, they would be French.
b. If Bizet and Verdi were compatriots, they would be Italian. (Quine 1950)

- Intuition: neither of these need be true.
- Way out: let the selection function select a set of closest worlds. $f(\llbracket \phi \rrbracket, w) \in \wp(W)$
- $M, w \models \phi > \psi$ iff the closest ϕ-worlds to w satisfy ψ.

Figure



Plural choice functions

Lewis 1972, Schlenker 2004

- "if-clauses as plural definite descriptions" of worlds
- "the extension to plural choice functions allows us to leave out the requirement that similarity or salience should always be so fine-grained as to yield a single "most salient" individual or a single "most salient" similar world"

The limit assumption

Suppose a line is 1 cm long. Take: "if this line were more than 1 cm long,...". According to Lewis, there need be no closest length to 1cm.

- Lewis's semantics (informally): $M, w \models \phi \mathrel{\Box\!\!\rightarrow} \psi$ iff some accessible $\phi\psi$-world is closer to w than any $\phi\neg\psi$-world, if there are any accessible ϕ-worlds.

Similarity models

- Similarity models: $M = \langle W, R, I, \{\leq_w\}_{w \in W}\rangle$, where $\leq_w$ is a centered total pre-order on worlds.
- centered total pre-order: transitive; total= $u \leq_w v \vee v \leq_w u$; centered: $i \leq_w w \Rightarrow i = w$
- The semantics (formally): $M, w \models (\phi \Box\!\!\rightarrow \psi)$ iff if $[\![\phi]\!] \cap R(w) \neq \emptyset$, then there is a $v \in R(w) \cap [\![(\phi \wedge \psi)]\!]$ such that there is no u such that $u \leq_w v$ and $u \in [\![(\phi \wedge \neg\psi)]\!]$.

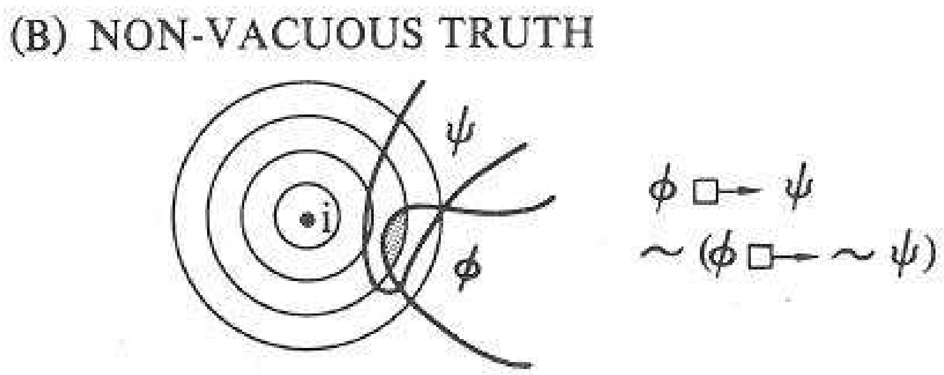
Comparative possibility

- Binary modality: $\phi \prec \psi$:="it is more possible that ϕ than ψ".
- $M, w \models (\phi \prec \psi)$ iff there exists $v \in R(w) \cap [\![\phi]\!]$ such that there is no u such that $u \leq_w v$ et $u \in [\![\psi]\!]$.
- $(\phi \Box\!\!\rightarrow \psi) =_{df} (\Diamond\phi \rightarrow ((\phi \wedge \psi) \prec (\phi \wedge \neg\psi))$

Similarity and Spheres

- A sphere around w is a set S of accessible worlds from w such that if $v \in S$, then for all u such that $u \leq_w v$, $u \in S$.
- $M, w \models (\phi \Box\!\!\rightarrow \psi)$ iff either there is no sphere S around w s.t. $[\![\phi]\!] \cap S \neq \emptyset$, or there is a sphere S around w s.t. $[\![\phi]\!] \cap S \neq \emptyset$ and for all $v \in S$, $M, v \models (\phi \rightarrow \psi)$.

Example



More: Girard 2006 on onions (=sphere systems)

Axiomatics

Lewis's VC

$(\phi \preceq \psi) =_{df} \neg(\psi \prec \phi)$ $\Diamond\phi =_{df} (\phi \prec \bot)$ $\Box\phi =_{df} \neg(\Diamond\neg\phi)$ $(\phi \Box\!\!\rightarrow \psi) =_{df} ((\Diamond\phi \rightarrow ((\phi \wedge \psi) \prec (\phi \wedge \neg\psi)))$
(PROP) All tautological schemata (MP) From ϕ and $(\phi \rightarrow \psi)$ infer ψ
$((\phi \preceq \psi \preceq \chi) \rightarrow (\phi \preceq \chi))$ (transitivity) $(\phi \preceq \psi) \vee (\psi \preceq \phi))$ (totality) $((\phi \preceq (\phi \vee \psi)) \vee (\psi \preceq (\phi \vee \psi)))$ (coherence)
(C) $((\phi \wedge \neg\psi) \rightarrow (\phi \prec \psi))$ (centering)
From $(\phi \rightarrow \psi)$ infer $(\psi \preceq \phi)$

Correspondence Lewis-Stalnaker

- Conditional Excluded Middle:
  VC+ $((\phi \Box\!\!\rightarrow \psi) \vee (\phi \Box\!\!\rightarrow \neg\psi))$ =C2
- No Uniqueness $\Rightarrow \not\models_{Lewis}$ *CEM*
- No Limit $\Rightarrow \not\models_{Lewis}$ *CEM*
- Uniqueness + Limit $\Rightarrow$ *CEM*
- Warning: Uniqueness alone $\not\Rightarrow$ *CEM*
  Model: let $W = [0, 1]$, let $V(p) = W$, let
  $V(q) = \{(\frac{1}{2})^n; n \geq 0\}$, let $u \leq_w v$ iff $u \leq v$.
  $0 \not\models (p \Box\!\!\rightarrow q) \vee (p \Box\!\!\rightarrow \neg q)$.

Comparisons and Perspectives

Which semantics is more adequate?

Lewis's semantics is more general than Stalnaker's, but it makes some disputable linguistic predictions.

The limit assumption

- Suppose Marie is shorter than Albert (5cm shorter). Suppose there are closer and closer worlds where Mary is taller than she is:

  (29) If Marie was taller than she is, she would (still) be shorter than Albert.

  Problem: there is a world where Mary is taller (e.g. by 1cm) where she is shorter than Albert, and that is closer to any world where she is taller and as least as tall as Albert.

The material conditional Stalnaker's logic Lewis's logic Comparisons and Perspectives

- In Stalnaker's system: problem averted since there has to be a closest world where Marie is taller than she is.
- Lewis's way out: count as equally similar all worlds in which Mary is taller up to 5cm. (weaken centering)
- "Coarseness may save Lewis from trouble, but it also saves the [plural] Choice Function analysis from Lewis" (Schlenker 2004)

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Negation

- Negation of the conditional is no longer conditional negation of the consequent:

  (30) $\Diamond\phi \nvDash \neg(\phi \Box\!\!\rightarrow \psi) \rightarrow (\phi \Box\!\!\rightarrow \neg\psi)$.

  If for every accessible $\phi\psi$-world there is a $\phi\neg\psi$-world at least as close, it does not follow that there is a $\phi\neg\psi$-world closer than any $\phi\psi$-world (Bizet case).

Limitations of both systems

Validities lost

Some of the “good" validities are lost in both Stalnaker's and Lewis's system:

- Import-Export: $\nvDash (\phi > (\psi > \chi)) \leftrightarrow (\phi \wedge \psi > \chi)$ (both directions)
- Simplification of Disjunctive Antecedents: $\nvDash (\phi \vee \psi > \chi) \rightarrow (\phi > \chi) \wedge (\psi > \chi)$
- Disjunctive Syllogism: $\phi \vee \psi \nvDash \neg\phi > \psi$

Examples from Natural Language

IE
a. If Mary leaves, then if John arrives, it won't be a disaster.
b. If Mary leaves and John arrives, it won't be a disaster.

SDA
a. If Mary or John leaves, it will be a disaster.
b. If Mary leaves, it will be a disaster, and if John leaves, it will be a disaster.

DS
The car took left, or it took right. Hence, if it did not take left, it took right.

What can be done?

- All such failures have been discussed: IE (McGee 1989), SDA (Alonso-Ovalle 2004, Klinedinst 2006), DS (Stalnaker 1975, see Lecture 5).
- The problem with all schemata: they all drive monotonicity back
- SDA: suppose $[\![\phi']\!] \subset [\![\phi]\!]$. Then $\phi \vee \phi' \equiv \phi$. If $\phi \vee \phi' > \chi \rightarrow \phi > \chi \wedge \phi' > \chi$, then $\phi > \chi \rightarrow \phi' > \chi$. (Klinedinst 2006).

The case of SDA

Klinedinst (2006: 127): *the problem is that what seems to be wanted is a semantics for conditionals that is both downward monotonic for disjunctive antecedents (at least in the normal case), but non-monotonic for antecedents in general.*

(31) If John had married Susan or Alice, he would have married Alice.

(32) If John had taken the green pill or the red pill – I don't remember which, maybe even both –, he wouldn't have gotten sick.

Klinedinst's proposal: the problem is pragmatic, and concerns our use of disjunction.

Summary: if P then Q

- Material Conditional: "not P and not Q".
- Singular Choice functions: "the closest P-world is a Q world".
- Plural Choice functions: "the closest P-worlds are Q-worlds".
- Similarity Ordering: "some *PQ*-world is closer than any $P\neg Q$-world"
- Strict Conditional: all P-worlds are Q worlds.

Comparisons

	Material	Stalnaker	Plural	Lewis	Strict
FA	Y	N	N	N	N
TC	Y	N	N	N	N
S	Y	N	N	N	Y
C	Y	N	N	N	Y
T	Y	N	N	N	Y
CEM	Y	Y	N	N	N
SDA	Y	N	N	N	Y
DS	Y	N	N	N	N
IE	Y	N	N	N	N

Bonus slides

Gibbard on IE

Theorem: Suppose (i), (ii) and (iii) hold: then $\Rightarrow$ and $\rightarrow$ are equivalent.

(i) $[\![(P \Rightarrow (Q \Rightarrow R))]\!] = [\![(P \wedge Q) \Rightarrow R]\!]$
(ii) $[\![(P \Rightarrow Q)]\!] \subseteq [\![(P \rightarrow Q)]\!]$
(iii) Si $[\![P]\!] \subseteq [\![Q]\!]$, alors $[\![(P \Rightarrow Q)]\!] = \top$

$[\![(P \rightarrow Q) \Rightarrow (P \Rightarrow Q)]\!] = [\![((P \rightarrow Q) \wedge P) \Rightarrow Q]\!]$
$[\![(P \rightarrow Q) \wedge P)]\!] = [\![(P \wedge Q)]\!]$
$[\![(P \rightarrow Q) \wedge P) \Rightarrow Q]\!] = [\![(P \wedge Q) \Rightarrow Q]\!]$
$[\![(P \wedge Q) \Rightarrow Q]\!] = \top$
$[\![(P \rightarrow Q) \Rightarrow (P \Rightarrow Q)]\!] = \top$
$[\![(P \rightarrow Q) \Rightarrow (P \Rightarrow Q)]\!] \subseteq [\![(P \rightarrow Q) \rightarrow (P \Rightarrow Q)]\!]$
$[\![(P \rightarrow Q) \rightarrow (P \Rightarrow Q)]\!] = \top$
$[\![(P \rightarrow Q)]\!] \subseteq [\![(P \Rightarrow Q)]\!]$

McGee on IE

If $f(A, w) \neq \lambda$, then:

$M, w \models_A p$ iff $I(\langle f(A, w), p\rangle) = 1$
$M, w \models_A \neg\phi$ iff $M, w \not\models_A \phi$
$M, w \models_A (\phi \wedge \psi)$ iff $M, w \models_A \phi$ et $M, w \models_A \psi$.
$M, w \models_A (\phi \vee \psi)$ iff $M, w \models_A \phi$ or $M, w \models_A \psi$.
$M, w \models_A (B \Rightarrow \phi)$ iff $M, w \models_{(A \wedge B)} \phi$

By def: $M, w \models \phi$ iff $M, w \models_{\top} \phi$

- Predictions:

1) $M, w \models A \Rightarrow \phi$ iff $M, w \models_{\top} A \Rightarrow \phi$ iff $M, w \models_A \phi$.

2) $M, w \models (A \Rightarrow (B \Rightarrow \phi))$ iff $M, w \models_A B \Rightarrow \phi$ iff $M, w \models_{(A \wedge B)} \phi$ iff $M, w \models (A \wedge B) \Rightarrow \phi$.

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Lecture 2. Conditionals as restrictors

Reminder on Stalnaker's semantics

Let us review Stalnaker's semantics for "if ϕ then ψ", the core of all other non-monotonic conditional semantics:

- Either ϕ holds at the actual world: $w \models \phi$, and in that case, $w \models \phi > \psi$ iff $w \models \phi \rightarrow \psi$ (no adjustment needed)
- Or $\neg\phi$ holds at the actual world: $w \models \neg\phi$, and so $w \models \phi > \psi$ iff $f(\phi, w) \models \psi$ (adjust w minimally to make it consistent with ϕ)

Are conditionals binary connectives?

- In all accounts we considered so far, we have assumed that the conditional is a binary connective. Yet compare:

  (1) Always, if it rains, it gets cold.

  (2) Sometimes, if it rains, it gets cold.

- Can the "if"-clause be given a uniform semantic role?

Conditionals and coordination (Bhatt and Pancheva 2006)

- If-clauses can appear sentence-initially and sentence-finally. Not so with *and* and *or*:

(3) a. Joe left if Mary left.
b. If Mary left Joe left.

(4) a. Joe left and Mary left.
b. *and Mary left Joe left.

- Even if, only if:

(5) a. Lee will give you five dollars even if you bother him.
b. *Lee will give you five dollars even and you bother him.

Lycan 2001

- Conjunction reduction

(6) a. I washed the curtains and turned on the radio
b. *I washed the curtains if turned on the radio.

- Gapping

(7) a. I washed the curtains and Debra the bathroom.
b. *I washed the curtains if Debra the bathroom

- VP-ellipsis:

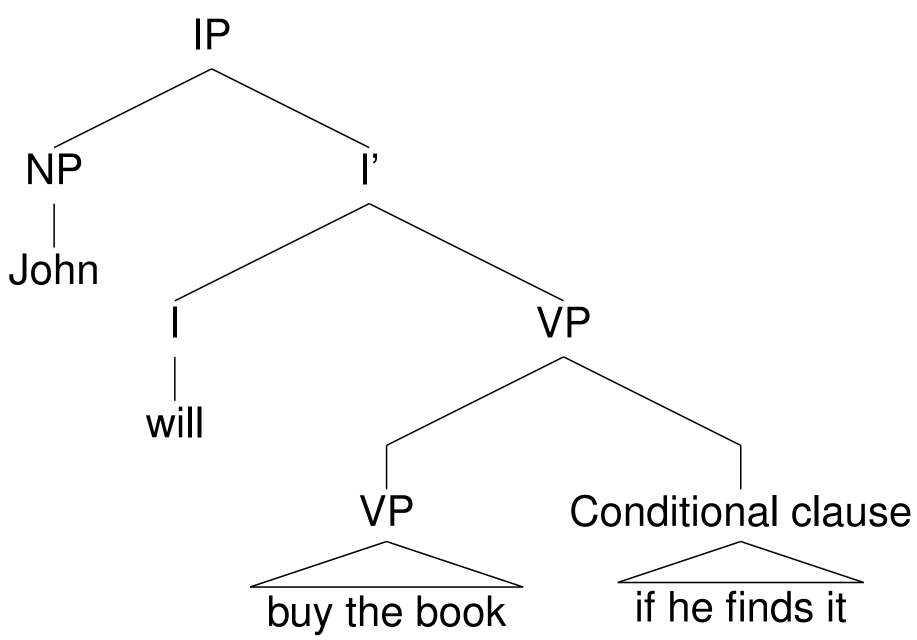
(8) I will leave if you do and John will ~~leave if you do~~, too

(9) I will leave if you do and John will *do so* too

“The data involving modification by only and even, and VP ellipsis phenomena provide strong evidence against the view that the antecedent and consequent of conditionals are coordinated. These data support the view that if-clauses are adverbials, like temporal phrases and clauses. Furthermore, pronominalization by then suggests that if-clauses are advervials, since their anaphoric reflex - then - is an adverb". Bhatt and Pancheva 2006

Syntactic representation

One possibility: from Haegeman 2003



Warning

The previous tree accounts only for the position of sentence-final if. The real story is more complicated (and left for syntacticians!):

> *Iatridou (1991) proposes that sentence-final if-clauses involve VP-adjunction, while sentence-initial if-clauses involve IP-adjunction. (Bhatt and Pancheva 2006)*

If-clauses as restrictors

A first try

(10) a. Always, if it rains, it gets cold.
b. $\forall t(R(t) \rightarrow C(t))$

(11) a. Sometimes, if it rains, it gets cold.
b. $\exists t(R(t) \rightarrow C(t))$.
c. $\exists t(R(t) \wedge C(t))$

Obviously, the intended reading is (11)-c, and (11)-b is simply inadequate. The intended reading is:

(12) Some cases in which it rains are cases in which it gets cold.

- Problem: how can “if" be given a uniform semantic role? What about other adverbs: *often*, *most of the time*,...?

Lewis 1975

- *If* as an adverbial restrictor

  the if of our restrictive if-clauses should not be regarded as a sentential connective. It has no meaning apart from the adverb it restricts (Lewis 1975: 14).

Kratzer 1991

"*The history of the conditional is the story of a syntactic mistake.*

- *There is no two-place if...then connective in the logical forms of natural languages.*
- *If-clauses are devices for restricting the domains of various operators.*
- *Whenever there is no explicit operator, we have to posit one."*
  (Kratzer 1991)

Monadic predicate logic with most

1. Atomic formulae: $Px, P'x, Qx, Q'x, \ldots$
2. Boolean formulae: $\phi := Px|\neg\phi|\phi \wedge \phi|\phi \vee \phi|\phi \rightarrow \phi$
3. Sentences: if ϕ, ψ are Boolean formulae: $\exists x\phi$, $\forall x\phi$, *Most* $x\phi$, as well as $[\exists x : \phi][\psi]$, $[\forall x : \phi][\psi]$, $[Most\ x : \phi][\psi]$.

Terminology: $[Qx : \phi(x)][\psi(x)]$:

- $\phi(x)$= quantifier restrictor
- $\psi(x)$ = nuclear scope

Semantics

Model: $M = \langle U, I \rangle$

1. $I(Px) \subseteq U$
2. $I(\phi \wedge \psi) = I(\phi) \cap I(\psi)$
   $I(\phi \vee \psi) = I(\phi) \cup I(\psi)$
   $I(\neg\phi) = \overline{I(\phi)}$
   $I(\phi \rightarrow \psi) = \overline{I(\phi)} \cup I(\psi)$
3. $M \models \forall x\phi$ iff $I(\phi) = U$
   $M \models \exists x\phi$ iff $I(\phi) \neq \emptyset$
   $M \models Most\ x\phi$ iff $|I(\phi)| > |I(\neg\phi)|$
4. $M \models [\forall x : \phi][\psi]$ iff $I(\phi) \subseteq I(\psi)$
   $M \models [\exists x : \phi][\psi]$ iff $I(\phi) \cap I(\psi) \neq \emptyset$
   $M \models [Most\ x : \phi][\psi]$ iff $|I(\phi) \cap I(\psi)| > |I(\phi) \cap \overline{I(\psi)}|$

The case of most

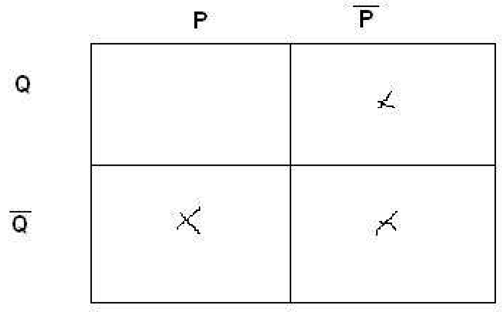
- $\models [\forall x : \phi][\psi] \leftrightarrow \forall x(\phi \rightarrow \psi)$
- $\models [\exists x : \phi][\psi] \leftrightarrow \exists x(\phi \wedge \psi)$

However:

- $\models Most\ x(\phi \wedge \psi) \rightarrow [Most\ x : \phi][\psi] \rightarrow Most\ x(\phi \rightarrow \psi)$
- But no converse implications.

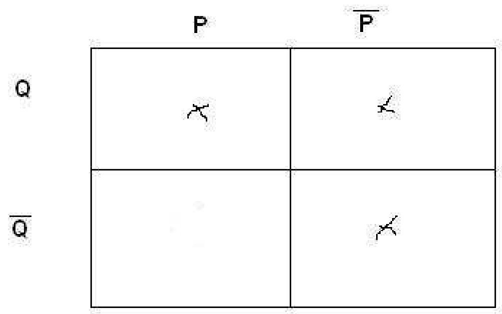
Most x are not P or Q $\not\Rightarrow$ Most Ps are Qs

- *Most* $x(Px \rightarrow Qx)$ is consistent with "no P is Q" (hence with $\neg[Most\ x : Px][Qx]$)



Most Ps are Qs $\nrightarrow$ Most x are P and Q

- $[Most\ x : Px][Qx]$ is consistent with $\neg Most\ x(Px \wedge Qx)$.



Restricted quantification

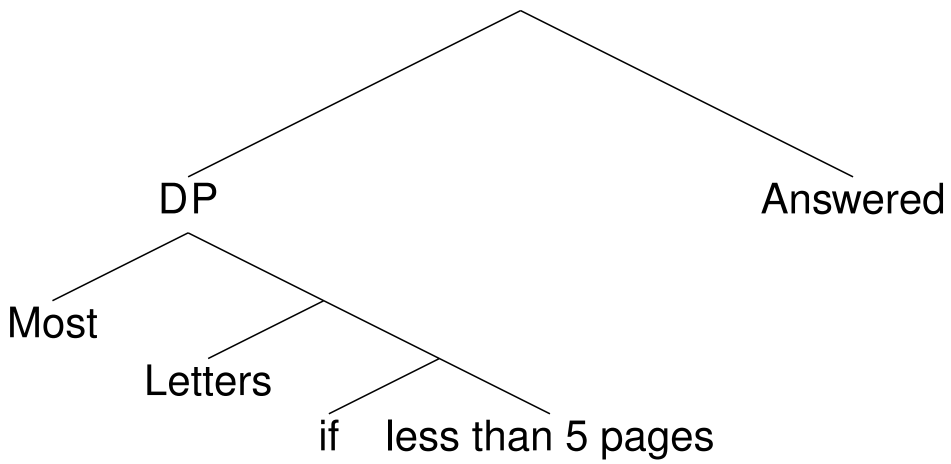
- Conclusion: the restrictor of <u>most</u> cannot be expressed by a material conditional (too weak) or a conjunction (too strong)
- Restricted quantification is needed to express if-clauses:

(13) a. Most of the time, if it rains, it gets cold.
b. $[\mathit{Most}\ t : Rt][Ct]$

(14) a. Most letters are answered if they are shorter than 5 pages.
b. Most letters that are shorter than 5 pages are answered.
c. $[\mathit{Most}\ x : Lx \wedge Sx][Ax]$.

Picture

- If-clauses usually attach a first quantifier restrictor, giving the effect of restrictive relative clauses.



Kratzer on conditionals

Conditional modality

How to analyze the interaction of a conditional with a modal?

(15) If a murder occurs, the jurors must convene. (in view of what the law provides)

Two prima facie candidates:

(16) a. $M \longrightarrow \Box J$
b. $\Box(M \longrightarrow J)$

Narrow scope analysis

Suppose that:

(17) a. No murder must occur. (in view of what the law provides)
b. $\Box\neg M$

Then, monotonicity problem:

(18) $\Box(\neg M \vee P)$ for every P.

In particular:

(19) $\Box(M \rightarrow J)$

Wanted: avoid automatic inference from "must $\neg p$" to "must if p, q" (a form of the paradox of material implication)

Wide scope analysis

(20) a. $M \rightarrow \Box J$
b. "no murder occurs, or the jurors must convene"

- Suppose a murder occurs. Then it should be an unconditional fact about the law that: $\Box J$, ie "the jurors must convene": too strong.

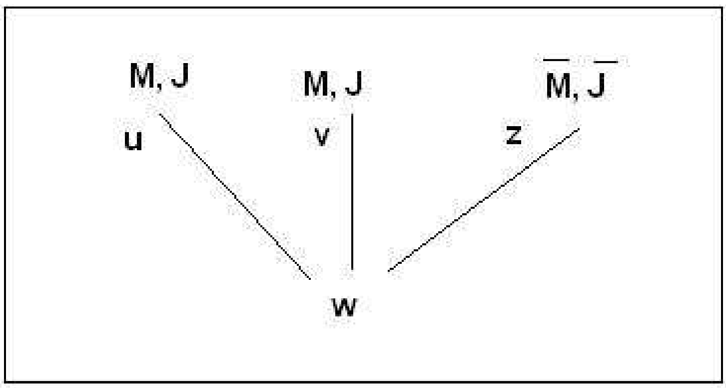
Illustration

Suppose there are two laws:

(21) a. If no murder occurs, the jurors are not allowed to convene.
b. $\neg M \rightarrow \Box\neg J$

(22) a. If a murder occurs, the jurors must convene.
b. $M \rightarrow \Box J$

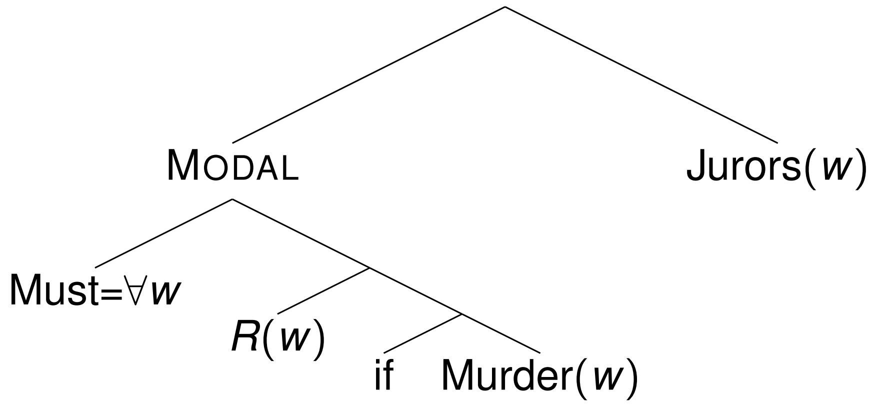
- Bad consequence: $\Box J \vee \Box\neg J$



- $M, w \not\models \Box J \vee \Box\neg J$, but intuitively: in all the worlds in which a murder occurs, the jurors convene, and in all the worlds in which no murder occurs, the jurors do not convene.

A syntactic improvement

(schema from von Fintel and Heim)



Problem: no semantic improvement on the strict conditional analysis when the modal is universal (“must") (but a semantic improvement with “might", “most of the time", ...)

Kratzer’s solution: informally

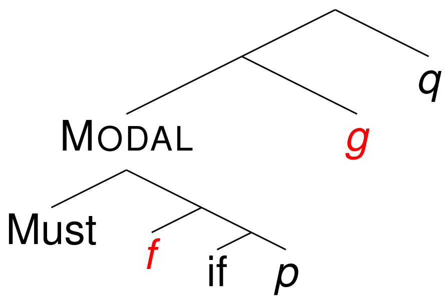
- $\Box p$: will be true not simply if p if true at all accessible worlds, but if p is true at all closest accessible worlds, or all ideal accessible worlds.
- Modality are doubly relative: to an accessibility relation, to an ordering.

Kratzer's solution: Doubly relative modalities

- Conversational background: *"a conversational background is the sort of entity denoted by phrases like what the law provides, what we know, etc. ... What the law provides is different from one possible world to another. And what the law provides in a particular world is a set of propositions...."*
- On Kratzer's analysis: 2 kinds of conversational backgrounds, modal base, and ordering source.

Schema

(from von Fintel and Iatridou 2004)



Correspondence with Stalnaker-Lewis:

- $f \equiv R(w)$
- $g \equiv \leq_w$

Modal base

- *The denotation of <u>what we know</u> is the function which assigns to every possible world the set of propositions we know in that world*
- Modal base: function f from W to $\wp(\wp(W))$ such that $f(w) = \{A, B, ...\}$
- By definition: $wR_f w'$ iff $w' \in \cap f(w)$

Ordering source

"there is a second conversational background involved... We may want to call it a stereotypical conversational background ("in view of the normal course of events"). For each world, the second conversational background induces an ordering on the set of worlds accessible from that world".

- Definition of an order: $\forall w, w' \in W, \forall A \subseteq \mathcal{P}(W) : w <_A w'$ iff $\{P; P \in A \wedge w' \in P\} \subset \{P; P \in A \wedge w \in P\}$
- A is a set of propositions: $w <_A w'$ iff w satisfies all propositions from A that w' satisfies and more besides.
- For each world, $g(w)$ picks such a set of propositions as an ordering source
- for $X \subseteq W$: $max_A(X)$=$\{w \in X | \neg\exists w' \in X : w' <_A w\}$

Kratzer's semantics for modals

- Model $M = \langle W, f, g \rangle$
- $w \models_{f,g} \Box\phi$ iff for all z such that $z \in max_{g(w)}(\cap f(w))$, $z \models_{f,g} \phi$
- ϕ is necessary iff it is true in all accessible worlds that come closest to the ideal.
- Remark: assumption that $<_{g(w)}$ always has some minimal elements (Limit Assumption)

Conditional modalities

- "if ϕ, then must ψ":=$(\Box : \phi)(\psi)$
- $w \models_{f,g} (\Box : \phi)(\psi)$ iff $w \models_{f',g} \Box\psi$, where $f'(w) = f(w) \cup \{ [\![\phi]\!]^{f,g} \}$.

"the analysis implies that there is a very close relationship between if-clauses and operators like must. They are interpreted together. For each world, the if-clause is added to the set of propositions the modal base assigns to that world. This means that for each world, the if-clause has the function of restricting the set of worlds which are accessible from that world".

Deontic conditional

(23) No murder must occur. If a murder occurs, the jurors must convene.

- Let: $g(w) = \{\neg M, M \rightarrow J\}$, and $f(w) = \emptyset$
- Let: $u \models \neg M$; $v \models M, J$; $z \models M, \neg J$. Then:

$$u <_{g(w)} v <_{g(w)} z$$

- $u \models_{f,g} \neg M$ and $\{u\} = max_{g(w)}(\cap f(w))$, so $w \models_{f,g} \Box \neg M$.
- $w \models_{f,g} (\Box : M)J$ iff $w \models_{f',g} \Box J$, where $f'(w) = f(w) \cup \{ [\![\text{M}]\!]^{f,g} \} = \{\{v, z\}\}$. Clearly $v \in \cap f'(w) = \{v, z\}$ satisfies J and belongs to $max_{g(w)}(\cap(f'(w))$.

Bare conditionals

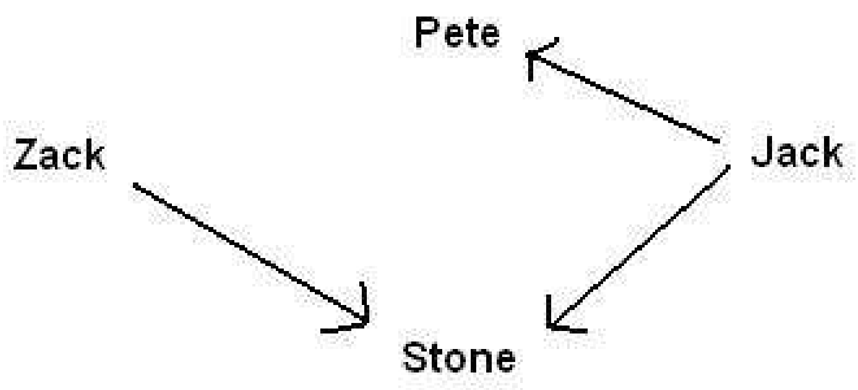
- According to Kratzer, bare conditionals are implicitly modalized
- Example discussed by Kratzer: epistemic modalities and conditionals.

Gibbard's riverboat example

Story (Gibbard 1981: 231): "Sly Pete and Mr. Stone are playing poker on a Mississipi riverboat. It is now up to Pete to call or fold. My henchman Zack sees Stone's hand, which is quite good, and signals its content to Pete. My henchman Jack sees both hands, and sees that Pete's hand is rather low, so that Stone's is the winning hand. At this point the room is cleared. A few minutes later, Zack slips me a note which says *if Pete called, he won*, and Jack slips me a note which says *if Pete called, he lost*.

- According to Kratzer: Zack's and Jack's utterances are both true.
- Why is it surprising? $\Diamond\phi \models_{Sta} (\phi > \psi) \rightarrow \neg(\phi > \neg\psi)$ (same with $\Box\!\!\rightarrow$ for Lewis).
- Each of them is true relative to a different modal base.

What Zack and Jack can see



A closer look

The difference is in Jack and Zack's modal bases

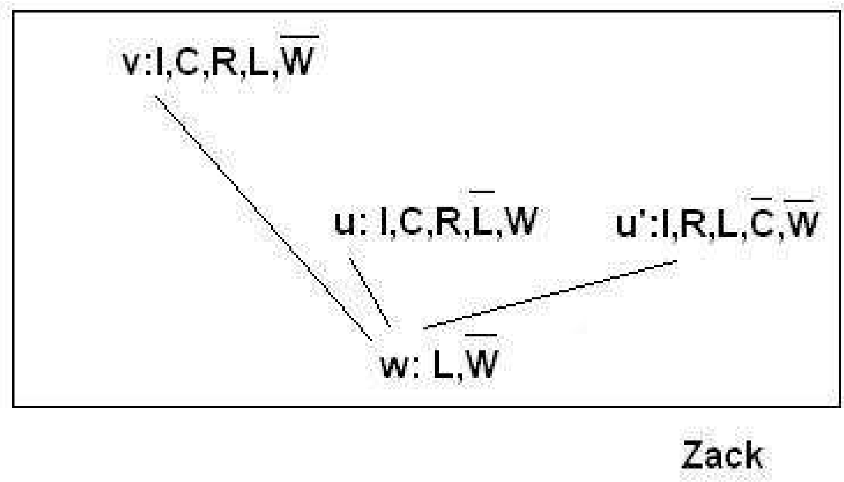
- $f_z(w) = \{$Pete is rational, Pete is informed about both hands$\} = \{R, I\}$
- $f_j(w) = \{$Pete is rational, Pete is informed about both hands, Pete's hand is lower than Stone's$\} = \{R, I, L\}$

A model for Sly Pete

Let: W=Pete will win.

- Common ordering source: $g_{j,z}(w) = \{RIC \rightarrow W, L \rightarrow \overline{W}\}$, ie nomic base includes: "a rational, informed player who calls wins, a lower hand does not win".
- For Zack: let $u \models I, C, R, \overline{L}, W$ and $v \models I, C, R, L, \overline{W}$, such that $\{u, v\} = \cap(f_z(w) \cup C)\}$. Then $u <_{g(w)} v$, and so $w \models_{f_z,g} (\Box : C)(W)$.
- For Jack: $\cap(f_j(w) \cup C)\} \subseteq L$ and does not contain any W-worlds. So $w \models_{f_j,g} (\Box : C)(\overline{W})$

Zack's state



- Note 1: we stipulate that no possible world is L and W, ie no logically impossible possible world.
- Note 2: note that $\Box(C \rightarrow W)$ would not hold at w with a strict conditional analysis.

Comparisons

Are “if"-clauses always restrictors?

von Fintel/Iatridou 2002

(24) a. Most but not all of the students will succeed if they work hard
b. Most but not all of the students who work hard will succeed.

Scenario: 4 students: a, b, c, d. Suppose that if a, b, c are to work hard, they will all succeed. Suppose however that only a and b actually work hard, and both succeed.

- Then (24)-a is true, (24)-b false.

Observations

According to Iatridou and von Fintel, such examples “do not fall under an extension of the Lewis-Kratzer analysis".

- Fintel’s analysis: a strict conditional analysis with variable domain restrictions.
- $Q(x)[R(x)][\text{ if} w\ P(x,w), Q(x,w)]$

However: even when “if" does not restrict the quantifier “most", it can continue to restrict a hidden quantifier (à la Kratzer) if one assumes a strict conditional analysis.

Conclusion: maybe the problem is purely syntactic.

A variant with “exactly"

(25) Exactly half of the students got an A if they worked hard

Claim: 2 interpretations here as well

(26) a. Exactly half of the students who worked hard got an A
b. $[1/2 : S \wedge W][A]$

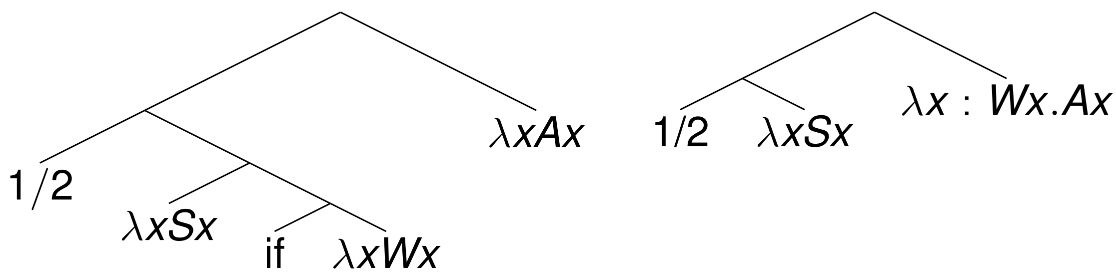
(27) a. Exactly half of the students got an A if those students worked hard.
b. $[1/2 : S][\text{if } W \text{ then } A]$.

Model

A model in which (26) is false and (27) true.

- $[\![S]\!] = \{a, b, c, d, e, f\}$
- $[\![W]\!] = \{a, b, c, d\}$
- $[\![A]\!] = \{a, b, c\}$.

Conditional as Presupposition



Let: $\lambda x : Wx.Ax$: function that takes value 1 if $Wx = Ax = 1$, value 0 if $Wx = 1$ and $Ax = 0$, undefined if $Wx = 0$.

“If" vs. “Necessarily if"

- Schlenker observes a contrast in monotonic behavior between:

(28) If the US got rid of all its weapons, there would be war. However, if the US and all other nuclear powers got rid of all their weapons, there should be peace.

(29) Necessarily, if the US got rid of all its weapons, there would be war. # However, if the US and all other nuclear powers got rid of all their weapons, there should be peace.

Modal strength

- Not necessarily an objection to Kratzer's thesis of implicit modalization: Kratzer can maintain that the strength of implicit modals is potentially lower than that of explicit modals:

(30) Ceteris paribus, if the US got rid of all its weapons, there would be war. But if the US and all nuclear powers got rid of all their weapons, there would be piece.

(31) Whatever happens, if the US got rid of all its weapons, there would be war. But if the US and all nuclear powers got rid of all their weapons, there would be piece.

- Strict conditionals drive monotonicity back.

Weak necessity

- At least good a possibility: $P \leq_{w,f,g} Q$ iff for all $u \in (\cap f(w) \cap Q)$ there is $v \in \cap f(w)$ such that $v \leq_{g(w)} u$ and $v \in P$.
- Better possibility: $P <_{w,f,g} Q$ iff $P \leq_{w,f,g} Q$ and not $Q \leq_{w,f,g} P$
- Weak necessity: $P <_{w,f,g} \overline{P}$
- $w_{f,g} \models \boxdot\phi$ iff $[\![\phi]\!] <_{w,f,g} \overline{[\![\phi]\!]}$

- Assume that relative to $g(w)$: $u \sim u' < v < z$, and $[\![\phi]\!] = \{u, v\}$, $[\![\neg\phi]\!] = \{u', z\}$. Then: $[\![\phi]\!] < [\![\neg\phi]\!]$. If u, u', v, z all belong to $\cap f(w)$, then ϕ is a weak necessity, but not a necessity (ie $w \not\models_{f,g} \Box\phi$).
- Clearly: possible to have $w \models_{f,g} (\boxdot : \phi)(\psi)$ and $w \not\models_{f,g} (\Box : \phi)(\psi)$.
- Benefit: treatment of expressions like “most likely"

Material conditional

- Let f such that for every w, $\cap f(w) = \{w\}$. (f is totally realistic= leaves no room for uncertainty)
- $w \models_{f,g} (\Box : \phi)(\psi)$ iff $\cap(f(w) \cup [\![\phi]\!]) \subseteq [\![\psi]\!]$.
- $\cap(f(w) \cup [\![\phi]\!]) = \{w\}$ if $w \models \phi$ and $= \emptyset$ if $w \not\models \phi$.
- $w \models_{f,g} (\Box : \phi)(\psi)$ iff $w \models \phi \wedge \psi$ or $w \not\models \phi$.

Strict conditional

- Suppose $f(w) = g(w) = \emptyset$ (all worlds accessible, all worlds equally close)
- For all $X \subseteq W$, $max_{\emptyset}(X) = X$
- Hence: $max_{\emptyset}(\cap(\emptyset \cup \{ [\![\phi]\!] \})) = [\![\phi]\!]$.
- $w \models_{\emptyset,\emptyset} (\Box : \phi)(\psi)$ iff $[\![\phi]\!]^{f,g} \subseteq [\![\psi]\!]^{f,g}$.

Probability

Explicit probability and conditionals

(32) The chances are n/m that if p then q

(33) The probability is n/m that if p then q

(34) If p, then the probability that q is n/m

Grice's Paradox

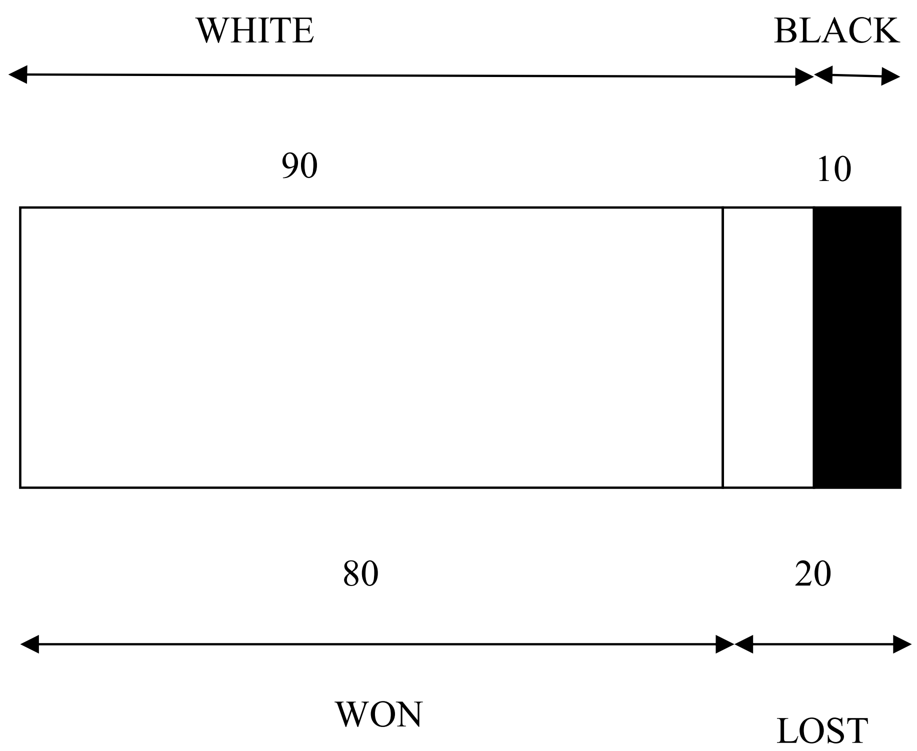
- Scenario: *Yog and Zog play chess. Up to now, there have been a hundred of games, no draws, and Yog had white 9 out 10 times. When Yog had white, he won 80 out of 90. When he had block, he lost 10 out of 10. We speak about the last game that took place last night and the outcome of which we do not know.*

(1) There is a probability of 8/9 that if Yog had white, he won

(2) There is a probability of 1/2 that if Yog lost, he had black

(3) There is a probability of 9/10 that either Yog didn't have white or he won

Yog and Zog's records



Grice's Paradox, cont.

- formalization:

(1) There is a probability of 8/9 that if Yog had white, he won

$$8/9(WHITE \Rightarrow WON)$$

(2) There is a probability of 1/2 that if Yog lost, he had black
$1/2(LOST \Rightarrow BLACK)$

$$1/2(\neg WON \Rightarrow \neg WHITE)$$

(3) There is a probability of 9/10 that either Yog didn't have white or he won

$$9/10(\neg WHITE \vee WON)$$

Intuitive semantics

- Suppose a finite set of worlds
- $M, w \models [n/m](A)$ iff $\frac{|A|}{|W|} = n/m$
- $M, w \models [n/m : A][B]$ iff $\frac{|AB|}{|A|} = n/m$
- Agreement with Kratzer's thesis
- Grice's paradox explained away

Summary

- if-clauses as restrictors: allows a uniform semantic treatment of if-clauses with all quantifiers (modal and temporal operators/generalized quantifiers over individuals)
- the Lewis-Kratzer thesis, however, is in fact orthogonal to the issue of monotonicity
- A derivation of the notion of similarity ordering (a version of premise semantics, more in Lecture 5)
- A prima facie smooth treatment of interaction with probability operators: more to come... (Lectures 3 and 4)

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Introduction to the Logic of Conditionals
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Lecture 3. Conditionals and Rational Belief Change

from semantics to epistemology

- during the last two lectures, we reviewed 5 semantical analyses of conditional sentences
- Lectures 3 and 4 will be much less linguistically oriented and will start from the epistemology of conditionals i.e. from rational belief attitudes towards conditionals
- more specifically, the main idea we will investigate: conditionals are closely linked to the dynamics of belief

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conditionals and belief dynamics

- the main questions we will address are the following:

✓ how to elaborate and formalize the idea that conditionals are linked to belief dynamics ?

✓ is this idea useful for the semantics of conditionals ?

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The Ramsey Test

the Ramsey Test

- the link between conditionals and the dynamics of belief is encapsulated in the famous **Ramsey Test (RT)**

  "If two people are arguing "If A will C ?" and are both in doubts as to A, they are adding A hypothetically to their stock of knowledge and arguing on that basis about C..." (F.P. Ramsey, 1929, "Law and Causality")

- Our reading of the (RT) : the belief attitude towards $(A \Rightarrow C)$ is determined by the belief attitude towards C after a rational belief change on A

the Ramsey Test, illustration

- consider

(OI) If Oswald did not kill Kennedy, someone else did it

- how to evaluate my degree of confidence in (OI) ?

  • first I suppose that Oswald did not kill Kennedy (contrary to my actual beliefs)
  • then I revise my belief state in a minimal and rational way
  • lastly I evaluate "Someone else did kill Kennedy" in my new belief state

scope of the Ramsey Test

- as we will see, the Ramsey Test is much debated ; but even for those who accept it, it is generally assumed that it is not valid for all conditionals. People who are confident in the Ramsey Test claim typically that it is OK for *indicative conditionals*

(OI) If Oswald did not kill Kennedy, someone else did it

(OS) If Oswald had not killed Kennedy, someone else would have

- If I add to my present state of beliefs the proposition that Oswald did not kill Kennedy, I am pretty sure that someone else did it. This thought experiment converges to my intuitive evaluation of (OI), not of (OS).

some complications

- a sentence problematic for the Ramsey Test (from R.Thomason):

  (1) If Reagan works for the KGB, I'll never find out

- but if I learned that Reagan works for the KGB, then I would have found out !
- several explanations/diagnoses:

  - conditionals and rational belief change go generally together, but there are complications with sentences dealing with belief attitudes

  - the Ramsey Test should be interpreted strictly: you have to suppose that the antecedent is true (not that it is true and that you *believe* that it is true !) (Stalnaker 1984)

two families of belief attitudes

- to elaborate formally on the Ramsey Test, one needs a model that captures both beliefs and rational belief change
- two kinds of (models of) beliefs:

(i) **full beliefs :** (a yes/no affair)
David believes that ϕ [acceptation]
David believes that $\neg\phi$ [reject]
David does not believe that ϕ [indeterminacy]

(ii) **partial beliefs :** (a matter of degree)
David believes that ϕ to degree *r*
David believes that it is probable that ϕ

two models of belief

- there are two main models of beliefand rational belief change :

(i) **belief revision theory** for full beliefs

(ii) **Bayesian probability** for partial beliefs

- *Ramsey Test for full beliefs* (RT_f) will be elaborated in the framework of belief revision (AGM etc.)
- *Ramsey Test for partial beliefs* (RT_p) will be elaborated in the framework of subjective probability. It will be equated (roughly) to a famous thesis, Adams Thesis (AT), equally named by Hajek & Hall (1994) the Conditional Construal of Conditional Probability (CCCP).

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the belief revision framework

- Ramsey Test in a dynamic framework for full beliefs
- full beliefs are represented by **beliefs sets** K i.e. sets of formulae that satisfy certain rationality constraints
- let the language be $\mathcal{L}_3^{\Rightarrow}$ = the **full conditional language** $\phi := p \mid \neg\phi \mid \phi \vee \phi \mid \phi \Rightarrow \phi$
- a relation of logical consequence $\models$ is given on $\mathcal{L}_3^{\Rightarrow}$ that extends the classical one

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the belief revision framework, cont.

- rationality constraints: belief sets (i) include all tautologies and (ii) are closed under $\models$
- the agent starts with an initial belief set K ; he or she receives a *message* represented by a formula ϕ and then revises his or her beliefs ; a new belief set $K * \phi$ is induced
- given a set of belief sets **K**, a **belief revision model** is a pair $(\mathbf{K}, *)$ where $* : \mathbf{K} \times \mathcal{L}_3^{\Rightarrow} \rightarrow \mathbf{K}$ is a revision operator (i.e. $K * \phi = *(K, \phi)$)

the Ramsey Test in belief revision

- in this framework, a weak reading of the Ramsey Test would be this one:

$$(\text{WRT}_f)\ A \Rightarrow C \in K \text{ if } C \in K * A$$

- the received reading (e.g. Gärdenfors 1986) is a stronger one:

$$\boxed{(\text{RT}_f)\ A \Rightarrow C \in K \text{ iff } C \in K * A}$$

The converse direction of (WRT_f) is called Conditional Driven Revision (CDR) by Bradley (2007) since it says that acceptation of a conditional commits to a revision policy

(CDR) and counterfactuals

- an example by Stalnaker (1984) shows that (CDR) is troublesome for *subjunctive conditionals*

(2) If Hitler had decided to invade England in 1940, Germany would have won the war ($A \Rightarrow C$)

(3) Hitler did decide to invade England in 1940 (A)

I give up $A \Rightarrow C$ rather than endorse C

rationality postulates

- (RT_f) won't give us anything interesting if we don't make assumptions on the revision operator
- the rationality constraints on $*$ have been intensively studied by belief revision theory
- examples:

  ✓ the postulate (K*2) $\phi \in K * \phi$ (Success)
  ✓ the postulate (K*5) $K * \phi = K_\perp$ iff $\models \neg\phi$ (Consistency)
  where $K_\perp$ is the absurd belief set that contains every sentences
- the paradigm view is the **AGM system** that includes 8 such postulates

the AGM postulates

(K*1) $K * \phi$ is a belief set
(K*2) $\phi \in K * \phi$
(K*3) $K * \phi \subseteq Cn\{K \cup \{\phi\}\} = K + \phi$
(K*4) If $\neg\phi \notin K$, $K + \phi \subseteq K * \phi$
(K*5) $K * \phi = K_\perp$ iff $\models \neg\phi$
(K*6) If $\models \phi \leftrightarrow \psi$, then $K * \phi = K * \psi$
(K*7) $K * (\phi \wedge \psi) \subseteq (K * \phi) + \psi)$
(K*8) If $\neg\psi \notin K * \phi$, $(K * \phi) + \psi \subseteq K * (\phi \wedge \psi)$

a logic for conditionals based on the Ramsey Test

- (RT_f) does not give us truth-conditions for $\Rightarrow$
- but we can base a *logic* on it by considering which formulae are valid in every belief revision system
- the logic will of course depend on the rationality postulates

  ✓ $A \Rightarrow A$ will e.g. be valid in b.r.m satisfying (RT_f) and (K*1): since $A \in K * A$, by (RT_f) $A \Rightarrow A \in K$.
- question: which logic is induced by the AGM system ?

from the Ramsey Test to Lewis and Stalnaker

- two of the AGM postulates are problematic for conditional logics (essentially (K*4) - more on this tomorrow)
- if we put these two postulates aside, the logic obtained is equivalent to Lewis system **VC** - see Lecture 1
- if we add the postulate according to which

  (K*C) If K is maximal (i.e. $\forall A$, A or $\neg A \in K$) then $K * \phi$ is maximal as well

  one obtains a logic equivalent to Stalnaker system **C2**
- a first convergence of the Ramsey Test and Lewis-Stalnaker logics
  (for an exhaustive survey of this literature, see Nute & Cross (2001))

Conditional probability

subjective probabilities

- the main model of partial beliefs and change of partial beliefs is the Bayesian probabilistic model
- two central tenets:

(1) static: the belief state of a rational agent at some time is represented by a probability measure - his or her degrees of beliefs obey the laws of probability

(2) dynamic: when a rational agent learns an information, he changes his doxastic state by the so-called rule of conditionalization (more on this later)

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probability functions

- let $\mathcal{L}$ be a language (a set of formulas) based (notably) on boolean connectives and $\models$ a consequence relation for that language (classical for boolean connectives)
- $P : \mathcal{L} \to \mathbb{R}$ is a **probability function** iff the following axioms are satisfied :

(P1) $0 \leq P(\phi) \leq 1$
(P2) If $\models \phi$, then $P(\phi) = 1$
(P3) If $\{\phi, \psi\} \models \bot$, then $P(\phi \vee \psi) = P(\phi) + P(\psi)$
(P4) If $\phi \equiv \psi$, then $P(\phi) = P(\psi)$

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probability distributions

- a probability function is a syntactic version of the usual notion of probability distribution defined on algebras of subsets
- Let $(W, \mathbf{E})$ be a measurable space. $p : \mathbf{E} \rightarrow \mathbb{R}$ is a **probability distribution** on $(W, \mathbf{E})$ if $\forall\, E, E' \in \mathbf{E}$

(P1') $0 \leq p(E) \leq 1$
(P2') $p(W) = 1$
(P3') If $E \cap E' = \emptyset$, then $p(E \cup E') = p(E) + P(E')$

- for a finite state space $W = \{w_1, ..., w_n\}$, one can see a probability distribution on $(W, 2^W)$ as a function $p : W \rightarrow \mathbb{R}$ s.t.

$$\sum_{w_i \in W} p(w_i) = 1$$

The Ramsey Test | Conditional probability | Adams Thesis | Adams Logic

linking the two notions

- semantics allows us to link these two notions. Given a model (W, I), consider $[\![\phi]\!] : \{w \in W : W, w \models \phi\}$. Let p be a probability distribution on W.
- then you may define $P(\phi) := p([\![\phi]\!])$ (the probability of ϕ is the probability of its truth)
- if I is a classical interpretation, then given an appropriate algebra $P(.)$ will obey (P1)-(P4)

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some basic properties of probability functions

- Let $\phi, \psi \in \mathcal{L}$;

(1) $P(\neg\phi) = 1 - P(\phi)$

(2) If $\models \neg\phi$, then $P(\phi) = 0$

(3) $P(\phi) + P(\psi) = P(\phi \vee \psi) + P(\phi \wedge \psi)$

(4) $P(\phi) = P(\phi \wedge \psi) + P(\phi \wedge \neg\psi)$ (Addition Theorem)

  - proof:
    (i) ϕ and $((\phi \wedge \psi) \vee (\phi \wedge \neg\psi))$ are equivalent
    (ii) $\phi \wedge \psi$ and $\phi \wedge \neg\psi$ are incompatible
    (iii) by (P3) and (P4), Addition Theorem

why should degrees of beliefs obey probabilities?

- the representation of a belief state by a belief set is coarse-grained but may seem much more intuitive than its representation by a probability function: **why should degrees of belief obey the laws of probability?**
- a first answer: think twice, it's self-evident !
- a more constructive answer: practical rationality requires beliefs as far as they guide action to obey probabilities = **Dutch Book Argument** Dutch Book

conditional probability

- the crucial notion in probabilistic belief dynamics is that of **conditional probability** $P(\psi|\phi)$
- $P(\psi|\phi)$ is defined (?) by the *Ratio Formula* (sometimes called the Quotient Rule):

$$P(\psi|\phi) = P(\phi \wedge \psi)/P(\phi) \text{ if } P(\phi) > 0$$

- **Fact**: for any probability function P, for any ϕ s.t. $P(\phi) > 0$, $P(.|\phi)$ is a probability function
- as a consequence, one can view
  • P as an initial or *a priori* doxastic state
  • $P(.|\phi)$ as an *a posteriori* doxastic state, that results from the learning ϕ - conditionalizing or learning by conditionalization

meaning of conditional probability

- the basic intuition behind the Ratio Formula : the probability of ψ given ϕ is the proportion of the ψ-worlds among the ϕ-worlds
- the meaning of the ratio formula can be grasped by considering the evolution of a world w after the information that ϕ is the case (i.e. that the actual world is among the $[\![\phi]\!]$-worlds):

(i) if $w \notin [\![\phi]\!]$ (i.e. $\delta_\phi(w) = 0$), $P(\{w\}|[\![\phi]\!]) = 0$

(ii) if $w \in [\![\phi]\!]$ (i.e. $\delta_\phi(w) = 1$), w's weight is normalized w.r.t. the total weight of $[\![\phi]\!]$-worlds :

$$P(\{w\}|[\![\phi]\!]) = P(\{w\})/\sum_{w' \in W}[\delta_\phi(w').P(\{w'\})]$$

an example

- initial probability P

w	ONE	TWO	THREE	FOUR	FIVE	SIX
P	1/6	1/6	1/6	1/6	1/6	1/6

- after conditionalization on $EVEN$, $P' = P(.|EVEN)$

w	~~ONE~~	TWO	~~THREE~~	FOUR	~~FIVE~~	SIX
P'	0	1/3	0	1/3	0	1/3

meaning of conditional probability, cont.

- **Fact**: if w and w' are compatible with ϕ, then

$$\frac{P(\{w\})}{P(\{w'\})} = \frac{P(\{w\}|\phi)}{P(\{w'\}|\phi)}$$

- this last property (which can be slightly generalized) gives a sense in which conditionalizing is a minimal way of changing (probabilistic) beliefs:

  "Conditionalizing P on A gives a minimal revision in this...sense: unlike all other revisions of P to make A certain, it does not distort the profile of probability ratios, equalities, and inequalities among sentences that imply A" (Lewis 1976)

conditionalization and invariance

- let's consider conditionalization as a probabilistic change rule of type $\mathbf{P} \times \mathcal{L} \rightarrow \mathbf{P}$
- an obvious property of conditionalization is **Certainty**: the new probability of the information is 1, $P_\phi(\phi) = 1$
- a less obvious property is **Invariance** of conditional probability: $\forall \psi \in \mathcal{L}$, $P(\psi|\phi) = P_\phi(\psi|\phi)$
- Jeffrey has noticed that Certainty + Invariance *characterize* conditionalization as a probabilistic change rule

conditionalizing as rational change of beliefs

- it is often claimed that conditionalizing is *the* rational way of changing one's partial beliefs
- the main kind of argument proposed to vindicate this claim is a *pragmatic justification* based on Dutch Books: it is named the *Dynamic Dutch Book* argument (Lewis - Teller)
- the DDB shows that if you don't update your beliefs by conditionalizing, then a clever bookie can devise a set of bets s.t. you will be willing to accept each of them whereas they collectively lead you to a sure loss

two theorems on conditional probabilities

- before coming back to conditionals, let me remind you two elementary theorems concerning conditional probabilities that will be useful in the sequel:

(i) **Bayes Theorem**:
$P(\psi|\phi) = [P(\phi|\psi).P(\psi)]/P(\phi)$

(ii) **Expansion By Case Theorem**:
$P(\phi) = P(\phi|\psi) \times P(\psi) + P(\phi|\neg\psi) \times P(\neg\psi)$
(follows from the Addition Theorem and the Ratio Formula)

Adams Thesis

Ramsey Test and Adams Thesis

- let's come back to the Ramsey Test and let P_ϕ denote a rational belief change based on the information that ϕ is the case
- a weak reading of (RT) for partial beliefs is this one: (WRT$_p$) If $P_A(C) = 1$, then $P(A \Rightarrow C) = 1$
- the received reading is a stronger one according to which

$$(\text{RT}_p)\ P(A \Rightarrow C) = P_A(C)$$

Ramsey Test and Adams Thesis, cont.

- the full quotation of (Ramsey, 1929) is nevertheless more specific:
  *"If two people are arguing "If p will q ?" and are both in doubts as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q ;... **they are fixing their degrees of belief in** q **given** p"*
- this specification of (RT$_p$) is known in the literature as **Adams Thesis** (E.W. Adams, *The logic of conditionals*, 1975) :
  "The fundamental assumption of this work is : the probability of an indicative conditional of the form "if A is the case then B is" is a conditional probability."

Adams Thesis

- Adams Thesis can be formulated in this way :

$$\text{(AT) } P(A \Rightarrow C) = P(C|A) \text{ if } P(A) > 0$$

- note that (AT) follows from (RT_p) + Conditionalization, but that it can be endorsed independently of any story about belief change
- (AT) is very compelling if we look at our intuitive judgements. Consider for instance the throw of a fair dice. What is the probability of

(4) if an even number comes up, the 6 comes up

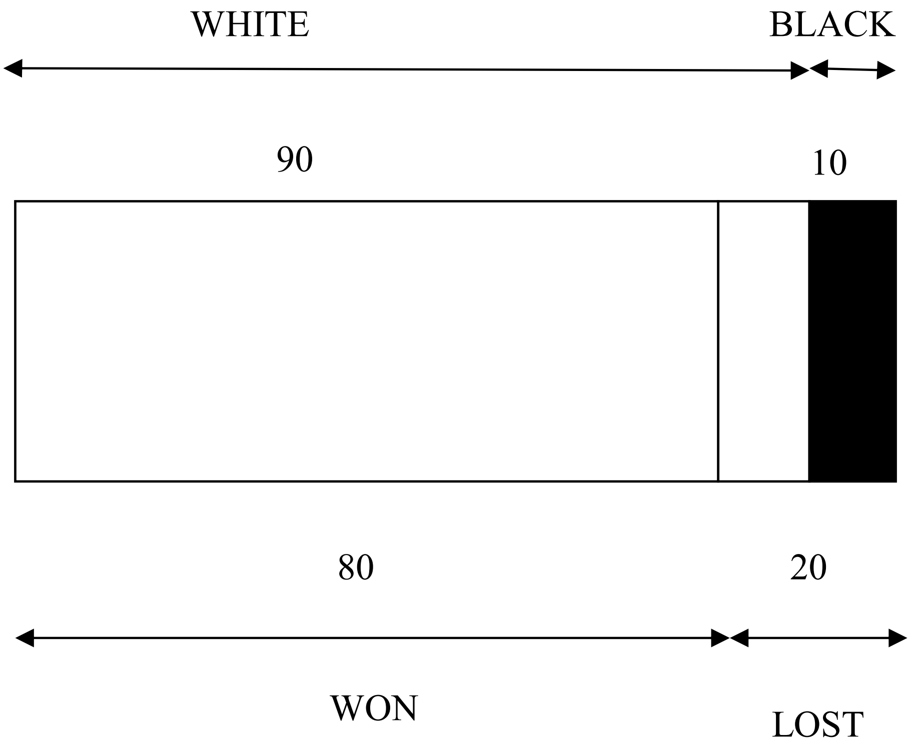
Grice's Paradox

- Scenario: *Yog and Zog play chess. Up to now, there have been a hundred of games, no draws, and Yog had white 9 out 10 times. When Yog had white, he won 80 out of 90. When he had block, he lost 10 out of 10. We speak about the last game that took place last night and the outcome of which we do not know.*

(1) There is a probability of 8/9 that if Yog had white, he won

(2) There is a probability of 1/2 that if Yog lost, he had black

Yog and Zog's records



material conditional and conditional probability

- it is easy to see how (AT) can contribute to the discussion on the semantics of the conditional. Consider the same example and the material conditional $(A \rightarrow C)$.

  What is the probability that either an even number doesn't come up or the 6 comes up ? 2/3

- the material conditional (1) doesn't conform to our intuitions on the probability of conditionals and (2) doesn't conform to Adams Thesis
- **Fact**: $P(\phi \rightarrow \psi) = P(\neg A) + P(A \wedge C) > P(\psi|\phi)$ except when $P(\phi \wedge \neg\psi) = 0$ or $P(\phi) = 1$ where $P(\phi \rightarrow \psi) = P(\psi|\phi)$

conditional probability and conjunction

- another consequence of (AT): $P(\phi \Rightarrow \psi) \geq P(\phi \wedge \psi)$ (strict inequality if $P(\phi) < 1$)
- if David believes strongly that $(\phi \wedge \psi)$, he believes at least as strongly that $(\phi \Rightarrow \psi)$
- to sum up:

$$P(\phi \wedge \psi) \leq P(\phi \Rightarrow \psi) \leq P(\phi \rightarrow \psi)$$

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overview

$$P(\phi \wedge \psi) \leq P(\phi \Rightarrow \psi) \leq P(\phi \rightarrow \psi)$$

- compare with the Stalnaker-Lewis conditional: $(\phi \wedge \psi) \models (\phi > \psi) \models (\phi \rightarrow \psi)$ therefore for any P,

$$P(\phi \wedge \psi) \leq P(\phi > \psi) \leq P(\phi \rightarrow \psi)$$

(see as well
$\models$ *Most* $x(\phi \wedge \psi) \rightarrow$ [*Most* $x : \phi][\psi] \rightarrow$ *Most* $x(\phi \rightarrow \psi)$)

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(AT) and Invariance

- remember: conditionalization is the belief change rule that obeys Certainty and Invariance of conditional probability
- putting this characterization together with (AT), this means that *the probability of a conditional $A \Rightarrow C$ doesn't evolve when you learn that the antecedent A is true* - the **invariance** of $\Rightarrow$
- not so for the material conditional: $P_A(A \rightarrow C) = P(C|A)) \leq P(A \rightarrow C)$ proof - you will necessarily lose confidence in the mat.cond when you learn that its antecedent is true !
- we will see later how F.Jackson exploits all this...to defend a truth-functional account of indicative conditionals

a counterexample to invariance

- invariance seems to be a compelling consequence of (AT). But see the following scenario (adapted from McGee 2000)
- you are quite confident in your banker who told you:

  (5) Don't be anxious. The prices won't continue to go down ($\neg A$). And (even) if they continue, this won't have much impact on you ($A \Rightarrow C$).

- then you learn that the prices continue to go down. So it seems that your banker is not reliable after all. As a consequence, your degree of credence in $A \Rightarrow C$ (if the prices continue to go down, this won't have much impact on you) decreases.
- so $P_A(A \Rightarrow C) < P(A \Rightarrow C)$!

(AT) and null-probability antecedents

- the conditional probability $P(\psi|\phi)$ is not defined when $P(\phi) = 0$
- this seems to be troublesome for *counterfactual* conditionals: for most of these, the antecedent is supposed to be known to be false
- two main reactions:

(i) modify something in Adams Thesis so that the modified Thesis can work even for null-probability antecedents (see Stalnaker 1970 based on Popper functions)

(ii) restrict Adams Thesis to indicative conditionals *and* consider that indicative conditionals are "zero-intolerant" i.e. do not really make sense when the antecedent is supposed to be false

why believe Adams Thesis ?

- main arguments

1. adequacy with intuitive judgments about probability of conditionals (look at the example of the fair dice).
(this is expected by the Lewis-Kratzer conditionals-as-restrictors view exposed yesterday, more on this tomorrow)
2. (AT) can be derived from (RT_p) + Conditionalization
3. Conditional Dutch Book more
4. from (AT), a logic of conditionals, **Adams logic**, can be built that represents quite faithfully the intuitively valid patterns of arguments involving conditionals

Adams Logic

Adams Logic: the language

- Let $\mathcal{L}_0$ be a propositional language (so-called **factual formulas**). The set of formulas $\mathcal{L}_1^{\Rightarrow}$ is given by
  - $F := p \mid \neg F \mid F \vee F$ (**factual formulas** = $\mathcal{L}_0$)
  - $\phi := F \mid F \Rightarrow F$ (**simple conditional formulas**)
- strong syntactic restriction: no embedding with conditionals, which appear only as main connectives.
- given a probability function P on $\mathcal{L}$ we know how to assign values to simple conditionals through (AT) and therefore how to extend P to $\mathcal{L}_1^{\Rightarrow}$.

Adams Logic: the language

- by contrast, (AT) does not tell us how to extend P defined initially on $\mathcal{L}$ to compounds of conditionals
  example : what is $P(B \wedge (A \Rightarrow C))$?
- if we were starting from a truth-conditional *semantics* for $\Rightarrow$ that assigns to each conditional $A \Rightarrow C$ a proposition $[\![A \Rightarrow C]\!]$ (e.g. Stalnaker-Lewis semantics), there would be no issue. In this case:
  $p([\![B \wedge (A \Rightarrow C)]\!]) = p([\![B]\!] \cap [\![A \Rightarrow C]\!])$
- caution: P extended to $\mathcal{L}_1^{\Rightarrow}$ is strictly speaking no longer a probability function
  For instance, it is not the case that
  $P(A \Rightarrow C) = P((A \Rightarrow C) \wedge B) + P((A \Rightarrow C) \wedge \neg B)$ since these two conjunctions are not in the language

probabilistic validity

- Adams develops a relation of logical consequence for $\mathcal{L}_1^{\Rightarrow}$ based on (AT)
- with (AT) alone - without truth-conditions for $\Rightarrow$ - one cannot develop the usual notion of Truth Preservation
- Adams idea: substituting **Probability Preservation** to Truth Preservation
- a second motivation (beside the lack of truth-conditions): outside mathematics, inferences are typically applied to less-than-certain premises

probabilistic validity

how to implement the Probability Preservation criterion ?

- option 1: **Certainty Preservation**: $\Gamma \models \phi$ iff $\forall P$, if $\forall \psi \in \Gamma$, $P(\psi) = 1$ then $P(\phi) = 1$ - this would fit the first motivation (we don't have truth-conditions for $\Rightarrow$) but not the second one (we typically reason from less-than-certain premises)
- option 2: **High Probability Preservation** criterion according to which $\Gamma \models \phi$ iff $\forall P$, ϕ is (roughly) at least as probable as the premises - this would fit the two motivations and this is the idea endorsed by Adams

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probabilistic validity, cont.

here is the official notion of p-validity (for "probabilistic validity"):

- $\psi \models_p \phi$ iff for no prob.func. P, $P(\phi) < P(\psi)$ iff for all P

$$U(\phi) \leq U(\psi)$$

  where $U(\phi) =_{df} 1 - P(\phi)$

- general case: $\Gamma \models_p \phi$ iff for all P

$$\boxed{U(\phi) \leq U(\psi_1) + ... + U(\psi_n)}$$

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probabilistic validity, cont.

- note that it can be the case that $\Gamma \models_p \phi$ and $P(\phi)$ is very low even if for each individual premise $\psi_i \in \Gamma$, $P(\psi_i)$ is very high
- this is as it should be. See for instance the **Lottery Paradox**: a lottery with one winning ticket among 100 available. Let ψ_i ($1 \leq i \leq 100$) mean "ticket $\sharp i$ is not the winning ticket" and let $\phi = \bigwedge_i \psi_i$
- $P(\psi_i) = 99/100$ but $P(\phi) = 0$

classical validity and *p*-validity

- **Proposition**: if $\Gamma \cup \{B\} \subseteq \mathcal{L}$ and $\Gamma \models_{PL} B$, then $\Gamma \models_p B$

  Proof:
  Assume $\{A_1, ..., A_n\} \models_{PL} B$
  (1) $\neg B \models_{PL} \neg A_1 \vee ... \vee \neg A_n$
  (2) $P(\neg B) \leq P(\neg A_1 \vee ... \vee \neg A_n)$ (since P is a probability function)
  (3) $P(\neg B) \leq P(\neg A_1) + ... + P(\neg A_n)$
  (4) $U(B) \leq U(A_1) + ... + U(A_n)$
  (5) $\{A_1, ..., A_n\} \models_p B$ ♠
- the converse is true as well; so for factual formulas, classical validity and *p*-validity coincide

$\neg A \therefore A \Rightarrow B$ is not p-valid

- failure of the paradox of the falsity of the antecedent

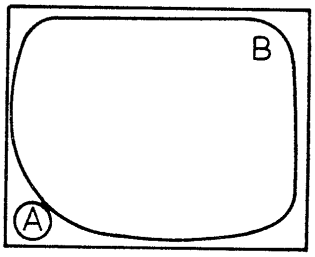


Fig. 2.

A = "It will not rain in Berkeley next year"
B = "It will rain in Berkeley next year"

$B \therefore A \Rightarrow B$ is not p-valid, cont.

- the same example can be used to illustrate the failure of the paradox of the truth of the consequent

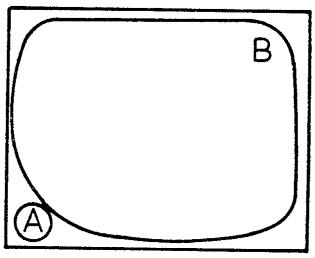


Fig. 2.

A = "It will not rain in Berkeley next year"
B = "It will rain in Berkeley next year"

$\rightarrow$ and $\Rightarrow$

- it is not the case that $A \rightarrow C \models_p A \Rightarrow C$
- but $P(A \rightarrow C) = 1$ implies $P(A \Rightarrow C) = 1$ (and conversely)
  If I am certain that the material conditional is true, I am certain that if A, C.
- clearly $P(A \rightarrow C) = 0$ implies $P(A \Rightarrow C) = 0$.
  $\rightarrow$ and $\Rightarrow$ collapse in the certain case.

$\{(A \Rightarrow B), (B \Rightarrow C)\} \therefore (A \Rightarrow C)$ is not p-valid

- failure of **Transitivity**:

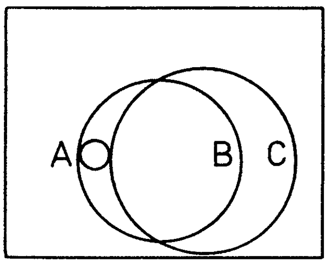


Fig. 5.

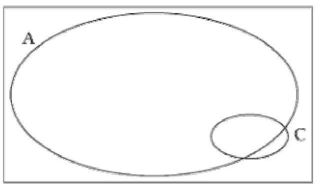
A = Smith will die before the election
B = Jones will win the election
C = Smith will retire after the election another example ?

$(B \rightarrow C) \therefore ((A \wedge B) \rightarrow C)$ is not *p*-valid

- but the following restriction is *p*-valid :
  $\{(A \Rightarrow B), ((A \wedge B) \Rightarrow C)\} \models_p (A \Rightarrow C)$
- the same diagram is a counterexample to **Antecedent Strenghtening**:
  ✓ $B \Rightarrow C$: If Jones wins the election, Smith will retire after the election
  × $(A \wedge B) \Rightarrow C$: If Smith dies before the election and Jones wins the election, Smith will retire after the election

$(A \Rightarrow \neg C) \therefore (C \Rightarrow \neg A))$ is not *p*-valid

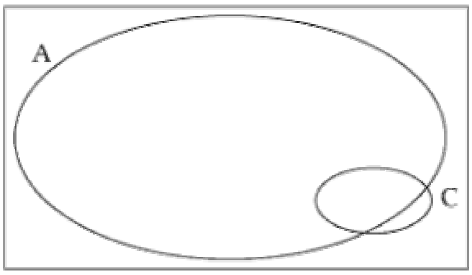
- failure of **Contraposition**: David is a movie critic and sees most of the movies. His friend Paul sees few movies, and most (but not all) of those he sees are seen by David as well. Let's consider a randomly chosen movie entitled *Life of a Logician*



It is likely that if David saw *Life of a Logician*, Paul didn't see it $(A \Rightarrow \neg C)$
It is unlikely that if Paul saw *Life of a Logician*, David didn't see it $(C \Rightarrow \neg A)$

$(A \vee C) \therefore (\neg A \Rightarrow C))$ is not p-valid

- failure of **Disjunctive Syllogism** (Bennett 2003):



A: There will be snow in Buffalo in 2009
C: A woman will be elected President of the USA in 2008

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p-validity and plausibility

- in Transitivity, Monotony, DS and Contraposition, *conclusions are conditionals* ; and in all our counter-examples *the probability of the conclusions' antecedents are very low*
- that's not a coincidence:
  (i) (ceteris paribus) the lower is the probability of the antecedent A, the more the probability of $A \Rightarrow C$ diverges from the one of $A \rightarrow C$
  (ii) these inferences are p-valid with $\rightarrow$
- by contrast, when $P(A)$ is high, these inferences will look perfectly acceptable since $U(A \Rightarrow C) \cdot P(A) = U(A \rightarrow C)$. This is how Adams explains that for indicative conditionals they may appear as OK in general

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a deductive system for Adams Logic

- here is a deductive system **ADS** for Adams Logic. A, B, C are factual formulas, ϕ, ψ are any formula

(R1) A premise ϕ can be stated on any line
(R2) If $A \vdash_{PL} B$, $\psi \therefore A \Rightarrow B$ for any ψ
(R3) $(\top \Rightarrow A) \therefore A$ and $A \therefore (\top \Rightarrow A)$
(R4) If $A \equiv_{PL} B$, $A \Rightarrow C \therefore B \Rightarrow C$
(R5) $\{A \Rightarrow C, B \Rightarrow C\} \therefore (A \vee B) \Rightarrow C$
(R7) $\{A \Rightarrow B, (A \wedge B) \Rightarrow C\} \therefore A \Rightarrow C$ (Restricted T)
(R8) $\{A \Rightarrow B, A \Rightarrow C\} \therefore (A \wedge B) \Rightarrow C$ (Restricted M)

- **Completeness Theorem**: $\Gamma \models_p \phi$ iff $\Gamma \vdash_{ADS} \phi$

Adams conditional and Stalnaker Conditional

- let us pause and compare (i) the inference patterns accepted and rejected by *p*-entailment and (ii) those accepted and rejected by Stalnaker semantics. They are pretty much the same !
- one of the most striking facts of the early contemporary research on conditionals is that *APL and Stalnaker Conditional coincide on their common domain* (Stalnaker 1970 ; Gibbard, 1980) : if $\Gamma \cup \{\phi\} \subseteq \mathcal{L}_1^{\Rightarrow}$ and Γ finite, then

$$\Gamma \models_p \phi \text{ iff } \Gamma \models_{Sta} \phi$$

idea of the proof

- the detailed proof of this impressive result is beyond the scope of this lecture
- if $\Gamma \models_p \phi$, then $\Gamma \models_{Sta} \phi$: can be proved by showing that Adams' deductive system is sound for Stalnaker semantics
- $\Gamma \models_p \phi$ if $\Gamma \models_{Sta} \phi$: much trickier ! Relies on Van Fraassen (1976) who shows how, given an algebra of factual propositions and a probability function on it, to extend it with conditional propositions in such a way that (1) the conditional obeys Stalnaker C2 logic and (2) (AT) holds.

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Summary and Perspectives

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summary

- the Ramsey Test and its rendering in belief revision (RT_f); the logic induced
- Adams Thesis (probability of conditionals = conditional probability)
- Adams Logic equivalent on their common domain to Stalnaker Logic **C2**

Kaufmann on Adams Thesis

- to end up we will present you a scenario by S.Kaufmann (*JPL*, 2004) which is intended to show that intuitive judgements about probabilities of conditionals does not (always) obey (AT).
- a colored ball is picked out of one of two bags X and Y

$P(X) = 1/4$	$P(Y) = 3/4$
10 red balls 9 of them with a black spot 2 white balls	10 red balls 1 of them with a black spot 50 white balls

- $R \Rightarrow B$: "If I pick a red ball, it will have a black spot"
- Question: what is the probability of $R \Rightarrow B$?

Kaufmann on Adams Thesis, cont.

- most people answer “low”. Here is a possible reconstruction of their reasoning:

$$
\begin{aligned}
P(R \Rightarrow B) \quad &= P(R \Rightarrow B|X)P(X) + P(R \Rightarrow B|Y)P(Y) \\
&\text{(Expansion by Case)} \\
&= P(B|RX)P(X) + P(B|RY)P(Y) \\
&\text{(Fact.Hypo)} \\
&= 9/10 \times 1/4 + 1/10 \times 3/4 = 0.3
\end{aligned}
$$

- we’ll come back to Fact.Hypo according to which, in full generality, $P(A \Rightarrow C|B) = P(C|A \wedge B)$

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Kaufmann on Adams Thesis, cont.

- but this is not conditional probability:

$$
\begin{aligned}
P(B|R) &= P(BR)/P(R) \\
&= P(B|RX)P(X|R) + P(B|RY)P(Y|R) \\
&= 9/10 \times 5/8 + 1/10 \times 3/8 = 0.6
\end{aligned}
$$

- main difference: $P(B|RX)$ is multiplied by $P(X)$ in the intuitive computation and by $P(X|R)$ in the cond.prob. computation. In the second case, one takes into account that the fact a red ball has been picked changes the probabilities of X and Y.

Kaufmann on Adams Thesis, cont.

- Let's call *local probability of a conditional*(P_l) the probability calculated in the same way as our intuitive computation. In general, for a partition $X_1, ..., X_n$, $P_l(A \Rightarrow C) = P(C|AX_1) \cdot P(X_1) + ... + P(C|AX_n) \cdot P(X_n)$
- Kaufmann claims that
  (i) the probability of indicative conditionals goes sometimes by local proba. (and not conditional proba.), contrary to (AT)
  (ii) belief change goes by conditional proba. ((i) and (ii) contradicts (RT_p))
  (iii) it can be rational to evaluate the probability of indicative conditionals by local proba.

Kaufmann on Adams Thesis, cont.

- I disagree with (iii) : I am inclined to think that people are wrong when following "local probability".
- Douven (forthcoming) shares this view and shows that local probability is inconsistent: the value of $P_l(A \Rightarrow C)$ is not invariant by the partition one considers.
- in Kaufmann's scenario he divides urn X in two sub-urns X_1 and X_2 and shows that $P_l(R \Rightarrow B)$ calculated with X_1, X_2, Y differs from its value with X and Y

Kaufmann on Adams Thesis, cont.

- there is still something puzzling in the scenario - that is connected with the Triviality Results we will present tomorrow
- $P(.|X)$ is a probability function so by (AT) and laws of probability, it should be that $P(R \Rightarrow B|X) = P(B|RX)$
- but given Expansion by Case, it follows that $P(R \Rightarrow B) \neq P(B|R)$ which contradicts (AT) !!
- so it is not so clear that (AT) gives us a rational way of evaluating conditionals either !

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- Proof of $P_A(A \rightarrow C) \leq P(A \rightarrow C)$

$P(A \rightarrow C|A) = P((\neg A \vee C) \wedge A)/P(A)$
$P(A \rightarrow C|A) = P(A \wedge C)/P(A) = P(C|A)$

back

- another example of the failure of Transitivity by Bennett (2003). *A farmer believes strongly (but not with certainty) that the gate into the turnip field is closed and that his cows have not entered that field.*
- He believes strongly
  ✓ $A \Rightarrow B$: if the cows are in the turnip field, the gate has been left open
  ✓ $B \Rightarrow C$: if the gate to the turnip field has been left open, the cows have not noticed the gate's condition
- But he does not believe (strongly)
  ✓ $A \Rightarrow C$: if the cows are in the turnip field, they have not noticed the gate condition

back

degrees of belief in factual sentences

- basic idea: my degree of belief p_A in A is the **betting price** i.e. fair price that I assign to the bet
  ✓ 1 euro if A (net gain: 1 - p_A)
  ✓ 0 if $\neg A$ (net gain: - p_A)
- more generally: my degree of belief p_A is s.t. $p_A.S$ is the fair price that I assign to the bet
  ✓ S euro if A (net gain: $(1 - p_A).S$)
  ✓ 0 if $\neg A$ (net gain: $-p_A.S$)
- it turns out (**Dutch Book Theorem**) that if your set of betting prices violate the laws of probability, then there exists a set of bets that you should accept and that result in a sure loss

back

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example

- assume that for Paul $p_A + p_{\neg A} > 1$. Then the bookie can devise two bets:

  ✓ B1: *on A* with stake 1

  ✓ B2: *on* $\neg A$ with stake 1

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an example, cont.

- here are the possible issues of the bets

w	B1	B2	Total Payoff
A	$(1-p_A)$	$-p_{\neg A}$	$1-p_A-p_{\neg A}<0$
$\neg A$	$-p_A$	$(1-p_{\neg A})$	$1-p_A-p_{\neg A}<0$

back

- what is my degree of belief in $A \Rightarrow C$?
- De Finetti (1937) introduces the notion of **conditional bet**
  ✓ S euro if AC (net gain: $(1-p).S$)
  ✓ 0 if $A\neg C$ (net gain: $-p.S$)
  ✓ called off if $\neg A$ (net gain: 0)
- does the fair price in a conditional bet reflects your degree of belief in $A \Rightarrow C$? If yes, then the **Conditional Dutch Book** shows that on pain of "incoherence", $p_{A\Rightarrow C} = \frac{p_{AC}}{p_A}$!

conditional Dutch Book

- suppose that $p_{A\Rightarrow C} < \frac{p_{AC}}{p_A}$. Then the bookie can device three bets:

  ✓ B1: *on AC* with stake p_A

  ✓ B2: *against A* with stake p_{AC}

  ✓ B3: *against* $A \Rightarrow C$ with stake p_A

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conditional Dutch Book, cont.

- here are the possible issues of the bets

w	B1	B2	B3
AC	$(1 - p_{AC}).p_A$	$-(1 - p_A)p_{AC}$	$-(1 - p_{A\Rightarrow C}).p_A$
$A\neg C$	$-p_{AC}.p_A$	$-(1 - p_A)p_{AC}$	$p_{\Rightarrow}.p_A$
$\neg A$	$-p_{AC}.p_A$	$p_{AC}.p_A$	0

- summing each line, one obtains as net results:

w	total payoff
AC	$p_{\Rightarrow}.p_A - p_{AC} < 0$
$A\neg C$	$p_{\Rightarrow}.p_A - p_{AC} < 0$
$\neg A$	0

back

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Lecture 4. Triviality Results and their implications

F.P. Ramsey

the Ramsey Test

- summary: we have seen two main ways of elaborating the so-called Ramsey Test, one in the belief revision framework (RT_f), the other in Bayesian probability (Adams Thesis)
- all this has to do with the **epistemology of conditionals**, even if we saw how it *could* have a strong impact on the **semantics of conditionals**
- it's time now to see how the Ramsey Test interacts with (i) other fundamental tenets of epistemology and (ii) semantics
- e.g.: ideally, from a semantic point of view, one would like truth-conditions for $\Rightarrow$ s.t. (AT) derives from these truth-conditions plus basic principles of probabilities ($p([\![A \Rightarrow C]\!]) = p([\![C]\!]|[\![A]\!])$)

triviality results

- at this point enter the "Bombshell"(s): an impressive sequence of **triviality results** intended to show basically that, on pain of triviality, the Ramsey Test cannot live peacefully with basic tenets of epistemology and/or semantics
- Menu:

(1) we will expose these triviality results

(2) we will then discuss the main *reactions* to these results (formal moves as well as lessons drawn from these) - mainly for the probabilistic results

Triviality Results 2 Ways Out No Truth Value Summary and Perspectives

Triviality Results

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Lewis Triviality Results (1976)

- the connective $\Rightarrow$ is **probability conditional** for a class $\mathbf{P}$ of probability functions iff for every $P \in \mathbf{P}$ and formulas A, C with $P(A) > 0$,

$$\text{(AT) } P(A \Rightarrow C) = P(C|A)$$

- the connective $\Rightarrow$ is a **universal probability conditional** iff (AT) holds for every probability function
- question: for a suitable (not necessarily universal) $\mathbf{P}$, does there exist a probability conditional ?

Triviality Results 2 Ways Out No Truth Value Summary and Perspectives

the factorization hypothesis

- the main assumption of LTR is the Factorization Hypothesis (FH):

$$\boxed{P(A \Rightarrow C|B) = P(C|AB), \text{ if } P(AB) > 0}$$

- there are two different lines of argument supporting (FH)

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factorization hypothesis and conditionalization

(i) **(FH) and conditionalization**: suppose that (AT) holds for every $P \in \mathbf{P}$ and that $\mathbf{P}$ is closed by conditionalization. Then (FH) holds for every $P \in \mathbf{P}$.

(1) $P(A \Rightarrow C|B) = P_B(A \Rightarrow C)$ (by closure)
(2) $P(A \Rightarrow C|B) = P_B(C|A)$ (by (AT) applied to P_B)
(3) $P(A \Rightarrow C|B) = P_B(C \wedge A)/P_B(A)$ (by Ratio Formula)
(4) $P(A \Rightarrow C|B) = P(C \wedge A \wedge B)/P(A \wedge B)$
(5) $P(A \Rightarrow C|B) = P(C|AB)$

factorization hypothesis and Import-Export

- **(FH) and the Import-Export Law (IE)**: suppose that (AT) holds for P. Then (FH) is equivalent to the probabilistic version of Import-Export

$$\text{(PIE) } P(B \Rightarrow (A \Rightarrow C)) = P(AB \Rightarrow C), \text{ si } P(AB) > 0$$

Proof: Supp. (PIE).

(1) $P(A \Rightarrow C|B) = P(B \Rightarrow (A \Rightarrow C))$ (by (AT))
(2) $P(A \Rightarrow C|B) = P(AB \Rightarrow C)$ (if $P(AB) > 0$ by (PIE))
(3) $P(A \Rightarrow C|B) = P(C|AB)$ (if $P(AB) > 0$ by (AT))
The other direction is analog. ♠

Lewis First Triviality Result

- **Theorem (LTR1):** Suppose that (AT) and (HF) holds for a class **P**. For any $P \in \mathbf{P}$, if $P(A \wedge C) > 0$ and $P(A \wedge \neg C) > 0$, then $P(C|A) = P(C)$
- *Proof:*
  (1) $P(A \Rightarrow C) = P(C|A)$ (AT)
  (2) $P(A \Rightarrow C|C) = P(C|A \wedge C) = 1$ (FH)
  (3) $P(A \Rightarrow C|\neg C) = P(C|A \wedge \neg C) = 0$ (FH)
  (4)$P(A \Rightarrow C) = P(A \Rightarrow C|C) \cdot P(C)$
  $+P(A \Rightarrow C|\neg C) \cdot P(\neg C)$ (expansion by case)
  (5) $P(C|A) = 1 \cdot P(C) + 0 \cdot P(\neg C) = P(C)$ (by (1)-(4)) ♠

example

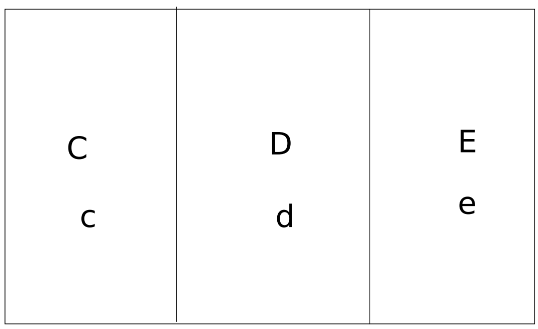
- the Result says that any two sentences A and C are probabilistically independent. But this excludes almost every probability function !
- example: the throw of a fair dice described by P. Let
  A="an even number comes up"
  C="the 6 comes up"
- the assumptions are satisfied : $P(A \wedge C) = \frac{1}{6}$ and $P(A \wedge \neg C) = \frac{1}{3}$
  But $P(C|A) = \frac{1}{3}$ whereas $P(C) = \frac{1}{6}$
- a probability function as simple and natural as P is therefore excluded from **P** !

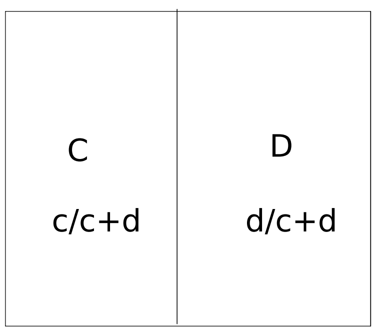
Lewis Second Triviality Result

- let's generalize from this example: suppose that C, D, E are three pairwise incompatible formulas, that each of them is possible given the semantics. If P assigns a positive weight to each of them, then $P(C|(C \vee D) \neq P(C)$ (C receives at least some of the weight of E)
- a **trivial language** is a language that does not contain such formulas.

Triviality Results 2 Ways Out No Truth Value Summary and Perspectives

non-trivial language





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Lewis Second Triviality Result

- LTR1 implies that "any language having a probability conditional is a trivial language"
- **Theorem (LTR2)** If (AT) and (FH) hold for P, then P assigns non-zero probabilities to at most two of any set of pairwise incompatible formulas
  (a trivial language is a sufficient but no necessary condition for this condition)

Lewis Third Triviality Result

- **Theorem (LTR3)** If (AT) and (FH) holds for P, then P takes at most four values.

  Proof: if P has more than four values, then there exists C and D s.t. $P(C) = x$ and $P(D) = y$ with $x + y \neq 1$. Hence $x \neq y$ and $(1 - x) \neq (1 - y)$. So $P(.)$ has at least 5 values. Suppose w.l.o.g. that $x + y < 1$. If $E = \neg C \wedge \neg D$, then $P(E) > 0$. H is (pairwise) incompatible with C and D.

  1. if C and D are incompatible, then C, D and E are all pairwise incompatible and non-zero weighted
  2. if $C \models D$, then C, $D \wedge \neg C$ and E are all pairwise incompatible and non-zero weighted (the same reasoning holds if $D \models C$)
  3. if C and D are not incompatible without one being the consequence of the other, then $C \wedge \neg D$, $D \wedge \neg C$ and E are all pairwise incompatible and non-zero weighted

what about the Stalnaker conditional?

- since $A > C$ is given truth-conditions (see Lecture 1), you may in principle perfectly deal with probabilities of Stalnaker conditional
- but if given a selection function f on W, $P(A > C)$ *is not* in general $P(C|A)$. A revision process called **imaging** by D.Lewis corresponds to $>$ i.e.

$$P^I_A(C) = P(A > C).$$

imaging

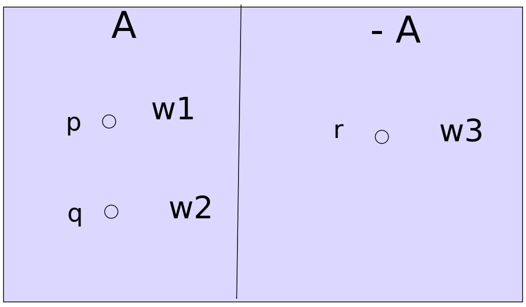
- the imaging rule is simple: the weight of a world w' excluded by the information that ϕ is wholly transferred to $f(\phi, w')$:

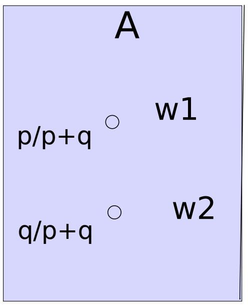
(i) if $w \notin [\![\phi]\!]$, $P^I_\phi(\{w\}) = 0$

(ii) if $w \in [\![\phi]\!]$, w keeps its initial weight *and* receives the weights of every world w' s.t. (i) $w' \notin [\![\phi]\!]$ and (ii) $w = f(\phi, w')$

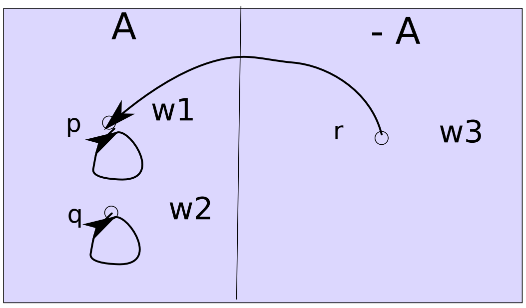
$$P^I_\phi(\{w\}) = \sum_{\{w' \in W : f(\phi, w') = w\}} P(\{w'\})$$

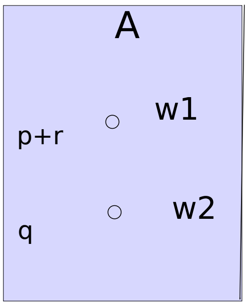
conditionalization on *A*





imaging on *A*





from Lewis to Gärdenfors

- to sum up: when we start from Adams construal of the Ramsey Test, we arrive at devastating triviality results
- maybe this construal is not good after all, and the issue disappears when we look at another framework...

Gärdenfors Triviality Result (1986)

- reminder: in a belief revision framework, the Ramsey Test means

$$(\text{RT}_f)\ A \Rightarrow C \in K \text{ iff } C \in K * A$$

- Gärdenfors's assumptions:

(K*2) $\phi \in K * \phi$ **(Success)**

(K*5w) if $K \neq K_{\perp}$ and $K * \phi = K_{\perp}$, then $\models \neg\phi$ **(Consistency)**

(K*P) if $\neg\phi \notin K$ and $\psi \in K$, then $\psi \in K * \phi$ **(Gärdenfors Preservation Condition)**
idea: don't give up beliefs unnecessarily !

Gärdenfors Triviality Result

- **Gärdenfors Triviality Result**: there is no non-trivial belief revision model (b.r.m.) that satisfies (RT_f) and (K*2), (K*5w) and (K*P).
- a b.r.m. is *non-trivial* iff there is at least 3 pairwise incompatible formulas and a belief set K which is consistent with these formulas ($\neg\phi_i \notin K$) (very close to Lewis definition)
- surprisingly, the proof doesn't rely directly on (RT_f) but on a consequence of (RT_f), **Monotonicity** (which doesn't involve conditionals):

  (K*M) $\forall K, K' \in \mathbf{K}$ and ϕ, if $K \subseteq K'$, then $K * \phi \subseteq K' * \phi$

proof

Bradley's Triviality Result

- let's come back to probabilities. Hajek & Hall (1994) (available upon request) provides an excellent overview of these results for 1976-1994
- two recent results by R. Bradley (2000, 2006) that rely on assumptions much weaker than (AT) (and (FH))
- **Bradley Preservation Condition**: for any $A, C \in \mathcal{L}$, if $P(A) > 0$ but $P(C) = 0$, then $P(A \Rightarrow C) = 0$

  [if A is supposed to be possible but C impossible, then the conditional "If A, then C" is supposed to be impossible]

Bradley Preservation Condition

- **Bradley Preservation Condition**: for any $A, C \in \mathcal{L}$, if $P(A) > 0$ but $P(C) = 0$, then $P(A \Rightarrow C) = 0$

  [if A is supposed to be possible but C impossible, then the conditional "If A, then C" is supposed to be impossible]
- example: the following epistemic situation violates Bradley Preservation Condition:

  (1) It might be the case that we go to the beach.

  (2) It is certain that we won't go swimming.

  (3) It might be the case that if we go to the beach, we will go swimming.
- Preservation Condition is implied by (AT) - but not conversely

Bradley Triviality Result

- relative to a consequence relation $\models$, a set of formulas $\mathcal{L}$ is **non-trivial** if it contains two factual sentences A, B and a simple conditional $A \Rightarrow B$ s.t. neither A nor $A \Rightarrow B$ implies B
- **Bradley Triviality Result 1**: if the Preservation Condition holds for every probability function on $\mathcal{L}$, then $\mathcal{L}$ is trivial.
- BTR1 relies on a simple property of probability distribution on (partially ordered) Boolean algebras: if $\neg Y \leq Z$ and $\neg X \leq Z$, there exists P on $(\Omega, \leq)$ s.t. $P(Y) > 0$, $P(Z) = P(X) = 0$.

Bradley Conservation Condition

- another consequence of (AT):

  Bradley Conservation Condition: for any $A, C \in \mathcal{L}$, if $P(A) > 0$ and $P(C) = 1$, then $P(A \Rightarrow C) = 1$

  [if A is supposed to be possible and C certain, then the conditional "If A, then C" is certain]

- **Bradley Triviality Result 2 (2006)**: Assume that Preservation & Conservation Conditions hold for every $P \in \mathbf{P}$ and that $\mathbf{P}$ is closed by conditionalization. If $P(A|C), P(A|\neg C) > 0$, then $P(A \Rightarrow C) = P(C)$

Bradley Triviality Result

- *Proof:*
  (1) $P(A \Rightarrow C) = P((A \Rightarrow C)|C) \cdot P(C)$
  $+P((A \Rightarrow C)|\neg C) \cdot P(\neg C)$ (expansion by case)
  (2) $P(C|C) = 1$ and $P(C|\neg C) = 0$ (standard laws of probability)
  (3) $P((A \Rightarrow C)|C) = 1$ (by Conservation and (2) since $P(A|C) > 0$
  (4) $P((A \Rightarrow C)|\neg C) = 0$ (by Preservation and (2) since $P(A|\neg C > 0$
  (5) $P(A \Rightarrow C) = P(C)$ (by (1), (3) and (4)) ♠

Lord Russell has been murdered

scenario: Lord Russell been murdered. Three suspects: the butler, the cook and the gardener. The butler did it probably (a motive and no alibi). The cook has no known alibi but no motive. The gardener has an alibi and no motive.

1. $P(\overline{C}) = 2/3$ [probably not the cook]
2. $P(\overline{B} \Rightarrow C) = 2/3$ [probably if not butler then cook]
3. $P(\overline{B} \Rightarrow G) = 1/3$ [improbable if not butler, gardener]
4. $P(\overline{B} \Rightarrow G|C) = 0$ [impossible, given cook, if not butler, gardener]
5. $P(\overline{B} \Rightarrow G|\overline{C}) = 1$ [certain, given not cook, if not butler, gardener]

Triviality Results 2 Ways Out No Truth Value Summary and Perspectives

Lord Russell has been murdered

- the probabilities:
  1. $P(\overline{C}) = 2/3$ [probably not the cook]
  2. $P(\overline{B} \Rightarrow C) = 2/3$ [probably if not butler then cook]
  3. $P(\overline{B} \Rightarrow G) = 1/3$ [improbable if not butler, gardener]
  4. $P(\overline{B} \Rightarrow G|C) = 0$ [impossible, given cook, if not butler, gardener]
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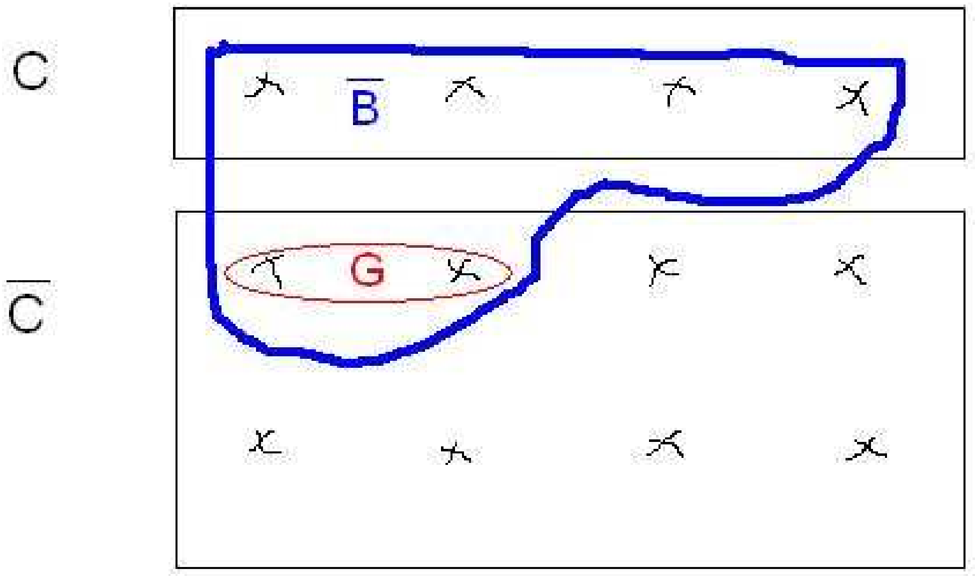
Lord Russell has been murdered

Yet intuitively: conditions 1-5 are jointly satisfiable.



Lord Russell has been murdered

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Lord Russell has been murdered

1. $P(\overline{C}) = 2/3$ [probably not the cook]
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Then

- $P(\overline{B} \Rightarrow G) = P(\overline{B} \Rightarrow G|C)P(C) + P(\overline{B} \Rightarrow G|\overline{C})P(\overline{C})$
- $P(\overline{B} \Rightarrow G) = 0 \cdot 1/3 + 1 \cdot 2/3 = 2/3$
- $1/3 = 2/3$: contradiction.

2 Ways Out

triviality: is there a way out?

- *if* (AT) is true *and* (indicative) conditionals have a classical truth-conditional semantics, then Lewis's assumptions are plausible and we end up with triviality

  "...we cannot have our cake and eat it, and we have to choose. If we accept the conditional probability thesis we have to give up truth-conditionality, and if we accept truth-conditionality, we have to give up the conditional probability thesis" (Adams, 1998)

Triviality Results 2 Ways Out No Truth Value Summary and Perspectives

triviality: which ways out?

- **OPTION 1:** give up (AT) and stick to a classical semantics for $\Rightarrow$ (Lewis, Jackson)
- **OPTION 2:** stick to (AT) but provide a non-classical semantics (Bradley, McDermott, Stalnaker & Jeffrey, McGee)
- **OPTION 3:** stick to (AT) and reject truth-conditions for $\Rightarrow$ (Bennett, Edgington, Appiah, Levi)

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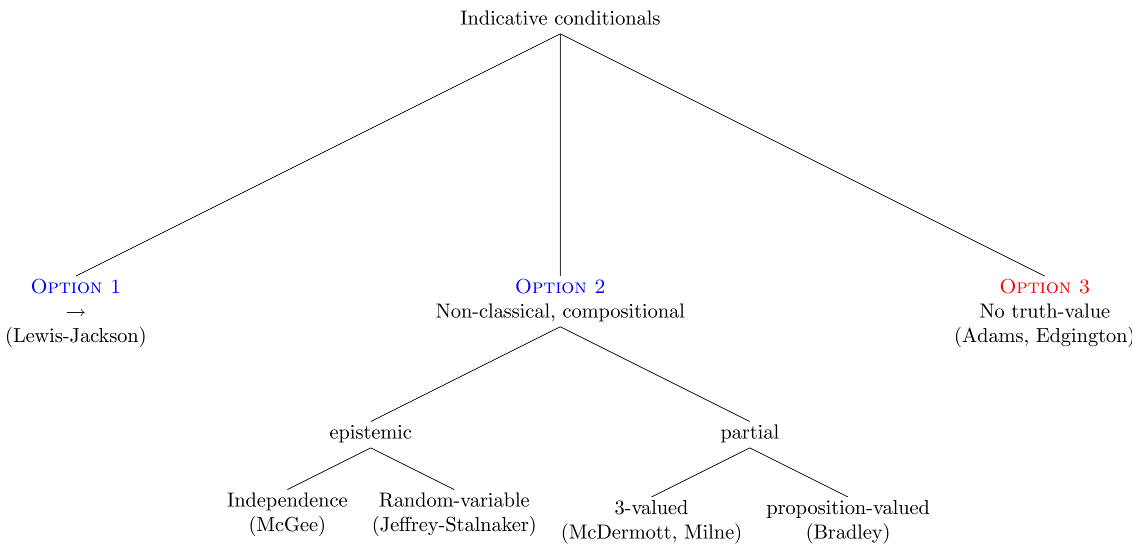

OPTION 1: good bye Adams

- **OPTION 1:** reject (AT) and stick to classical semantics for $\Rightarrow$
- some give up (AT) (or the Ramsey Test in general): position endorsed by Gärdenfors for (RT_f) and by Jackson (2006, MS.) for (AT): "Our usage of the indicative conditional construction is governed by a mistaken intuition"
- main elaboration of OPTION 1 due to Lewis (1976) and Jackson (1979): truth-functional semantics for $\Rightarrow$ + substitute for (AT) = the truth-functional view

the truth-functional view: Lewis & Jackson

- here are the main tenets of the truth-functional view (or "New Horseshoe Theory" - Lycan):
- indicative conditionals have truth-conditions
- the truth-conditions of indicative conditionals are those of the material conditional
- the probability of **truth** of an indicative conditional is the probability of the corresponding material conditional
- the **assertability** of an indicative conditional goes (nonetheless) by conditional probability (substitute for (AT))
- the divergence between assertability and probability of truth is explained by some pragmatic principle

Triviality Results 2 Ways Out No Truth Value Summary and Perspectives

conditional probability and robustness

- question : how to derive "assertability goes by conditional probability" a from pragmatic principle ?
- Jackson's theory (1979): **robustness**: when someone asserts "If A, then C", he believes strongly that $A \rightarrow C$ *and* indicates that his or her belief is robust with respect to the antecedent i.e. his or her belief in the conditional would still be strong were he or she learns that the antecedent is true
- this theory predicts that the assertability of $(A \Rightarrow C)$ depends on $P(A \rightarrow C|A)$
  $P(A \rightarrow C|A) = P(\neg A \vee C|A) = P(AC|A) = P(C|A)$

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why robustness ?

- what is the point of robustness ?
- Jackson's explanation: robustness w.r.t. the antecedent guarantees the use of Modus Ponens.
- in general, if you believe $(A \rightarrow C)$, don't know about A and C and are interested by C, you will inquire whether A. But evidence for A can be evidence against $(A \rightarrow C)$. If this is so, you won't be in a position to infer C by MP.
- not so if robustness w.r.t. the antecedent

objections to Lewis-Jackson

- objection 1 (e.g. Bradley 2002): it is not only the degree to which we are disposed to assert a conditional that equates the conditional probability but the degree to which we believe it
- objection 2: embeddings
  (i) embbedings of material conditionals are problematic:

  (4) If John is in Paris, he is in France

  (5) Either if Jones is in Paris, he is in Turkey, or if John is in Istanbul, he is in France

  (ii) how to explain non-semantically the trouble with this kind of inference ? how to extend the basic explanation beyond simple conditionals?

objections to Lewis-Jackson, cont.

- objection 3 (Edgington): the analogy between $\Rightarrow$ vs. $\rightarrow$ and "but" vs. "and" often invoked by Jackson to motivate his theory is not satisfactory:
  we would say that

  (6) Paul is French but smart

  is probably true but inappropriate ; not so for

  (7) If Jacques Chirac is elected President for the third time, he will double income tax

OPTION 2: let's have the cake and eat it !

- **OPTION 2:** stick to (AT) but provide a non-classical semantics
- two main alternatives:

(a) either a belief-independent non-classical semantics (coined thereafter **partial semantics** for reasons that will become clear), or

(b) a belief-dependent non-classical semantics (coined **epistemic semantics**)

McGee Logic (MGL)

- McGee (1989) "Conditional Probabilities and Compounds of Conditionals" is the main existing attempt to develop Adams's work in the face of Lewis's Triviality Results
- McGee allows the embedding of conditionals in the consequents of conditionals: conditionals have the general form

$$(A \Rightarrow \phi)$$

where A is a factual formula and there is no restriction on ϕ. Let $\mathcal{L}_2^{\Rightarrow}$ denote MGL language:
- $F := p \mid \neg F \mid F \vee F$ (= $\mathcal{L}_0$)
- $\phi := F \mid F \Rightarrow \phi$
($\mathcal{L}_0 \subseteq \mathcal{L}_1^{\Rightarrow} \subseteq \mathcal{L}_2^{\Rightarrow} \subseteq \mathcal{L}_3^{\Rightarrow}$)

Triviality Results 2 Ways Out No Truth Value Summary and Perspectives

McGee Logic (MGL)

- McGee's main assumptions:

  ✓ the standard axioms of probability functions (P1)-(P4)
  ✓ the Import-Export Law ($P(B \Rightarrow (A \Rightarrow C)) = P(AB \Rightarrow C)$, if $P(AB) > 0$)
  ✓ Adams Thesis restricted to *simple conditionals*
- instead of the full Adams Thesis, McGee requires a principle called the **Independence Principle**

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McGee Logic (MGL)

- (simple) **Independence Principle** : if A and C are classically incompatible and $P(A) > 0$, then

$$P(C \wedge (A \Rightarrow B)) = P(C).P(A \Rightarrow B)$$

which implies that the probability of $(A \Rightarrow B)$ should not change on the assumption that the antecedent is false

why?

- example:
$P(ODD \wedge (EVEN \Rightarrow SIX) = P(ODD) \cdot P(EVEN \Rightarrow SIX)$
- under the assumption that $P(A \wedge (A \Rightarrow B)) = P(A \wedge B)$, it implies Adams Thesis for simple conditionals proof

sketch of the machinery

- we saw in Lecture 3 that (AT) allows us to extend P defined on $\mathcal{L}_0$ to simple conditionals ($\mathcal{L}_1^{\Rightarrow}$)
- the Independence Principle (+ Modus Ponens for factual formulas) allows us to extend P on $\mathcal{L}_0$ to Boolean compounds of factual formulas and simple conditionals
- Import-Export allows us to extend P to right-nested conditionals hence to $\mathcal{L}_2^{\Rightarrow}$

first example: $P(B \wedge (A \Rightarrow C))$

- what is $P(B \wedge (A \Rightarrow C))$?
- intuitive idea:
  - when $\neg B$, the whole is false
  - when ABC, the whole is true
  - when $AB\neg C$, the whole is false
  - when $\neg AB$, the value is $P(A \Rightarrow C)$
- $P(B \wedge (A \Rightarrow C)) = P(ABC) + P(\neg ABC) \cdot P(A \Rightarrow C)$

first example: $P((A \Rightarrow B) \wedge (C \Rightarrow D))$

- what is $P((A \Rightarrow B) \wedge (C \Rightarrow D))$?
- intuitive idea:
  - when $ABCD$, the whole is true
  - when $AB\neg C$, the value is $P(C \Rightarrow D)$
  - when $\neg ACD$, the value is $P(A \Rightarrow B)$

  and all is normalized by $P(A \vee C)$
- $P((A \Rightarrow B) \wedge (C \Rightarrow D)) = 1/P(A \vee C) \cdot$
  $[P(ABCD)$
  $+P(\neg ACD) \cdot P(A \Rightarrow B)$
  $+(P(AB\neg C) \cdot P(C \Rightarrow D)]$

McGee and Lewis

- let's have a look at how McGee deals with Lewis trivialization:
  (1) $P(A \Rightarrow C) = P(C|A)$ (AT)
  (2) $P(A \Rightarrow C|C) = P(C|A \wedge C) = 1$ (FH)
  (3) $P(A \Rightarrow C|\neg C) = P(C|A \wedge \neg C) = 0$ (FH)
  (4)$P(A \Rightarrow C) = P(A \Rightarrow C|C) \cdot P(C)$
  $+P(A \Rightarrow C|\neg C) \cdot P(\neg C)$ (expansion by case)
  (5) $P(C|A) = 1 \cdot P(C) + 0 \cdot P(\neg C) = P(C)$ (by (1)-(4)) ♠
- (FH) is not valid ; since (PIE) is, this implies that in general $P(B \Rightarrow (A \Rightarrow C)) \neq P(A \Rightarrow C|B)$

MGL and Modus Ponens

- a surprising feature of MGL: Modus Ponens is not valid ! (it is valid only for simple conditionals)
- McGee's scenario (1985):
  3 candidates in 1980 presidential campaign, Reagan (Republican, ahead in the polls), Carter (Democrat, second) and Anderson (Republican, distant third)
  R : "Reagan will win the election"
  A : "Anderson will win the election"
- the problematic inference:
  $((R \vee A) \Rightarrow (\neg R \Rightarrow A))$
  $(R \vee A)$
  $\therefore (\neg R \Rightarrow A)$

Stalnaker & Jeffrey (1994)

- Stalnaker & Jeffrey (1994) propose *"an embedding of Adams' treatment of the simplest conditionals in a more permissive or comprehensive framework allowing arbitrary embeddings of conditional sentences within each other and within truth functional compounds."*
- idea: the semantic value of a factual formula A can be seen as a function from possible worlds to truth value and therefore as a (degenerate) random variable δ_A
- the probability of a factual formula can be seen as the expectation of its random variable: $P(A) = \mathbf{E}(\delta_A)$ (weighted average of the values of δ_A on possible worlds)

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Expectation-Based Adams Thesis

- you can see in general the semantic value of a sentence ϕ as a random variable δ_ϕ
- new formulation of Adams Thesis = **Expectation-Based Adams Thesis (EBAT)** :

$$\mathbf{E}(\delta_{A \Rightarrow \phi}) = \mathbf{E}(\delta_\phi | \{w : \delta_A(w) = 1\})$$

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an example from Edgington (2006)

- (EBAT) : $\mathbf{E}(\delta_{A\Rightarrow\phi}) = \mathbf{E}(\delta_\phi | \{w : \delta_A(w) = 1\})$
- scenario: 50% of the balls are red (R), 80% of the red balls have a black spot (B)
- the semantic value of $R \Rightarrow B$ is a random variable $\delta_{R\Rightarrow B}$ defined on W:

✓ $\delta_{R\Rightarrow B}(w) = 1$ if $w \in [\![R \wedge B]\!]$
✓ $\delta_{R\Rightarrow B}(w) = 0$ if $w \in [\![R \wedge \neg B]\!]$
✓ $\delta_{R\Rightarrow B}(w) = P([\![B]\!]|[\![R]\!]) = 8/10$ if $w \in [\![\neg R]\!]$

$\mathbf{E}(\delta_{R\Rightarrow B}) = P([\![R \wedge B]\!]) \cdot 1 + P([\![R \wedge \neg B]\!]) \cdot 0 + P([\![\neg R]\!]) \cdot 8/10$
$\mathbf{E}(\delta_{R\Rightarrow B}) = 4/10 + 5/10 \cdot 8/10 = 8/10 = P([\![B]\!]|[\![R]\!])$!

- this is not a coincidence !

$\mathbf{E}(\delta_{R\Rightarrow B}) = P([\![R \wedge B]\!]) \cdot 1 + P([\![R \wedge \neg B]\!]) \cdot 0 + (1 - P([\![R \wedge B]\!]) - P([\![R \wedge \neg B]\!])) \cdot P([\![B]\!]|[\![R]\!])$

$\mathbf{E}(\delta_{R\Rightarrow B}) = P([\![B]\!]|[\![R]\!]) + P([\![R \wedge B]\!]) - P([\![R \wedge B]\!]) \cdot P([\![B]\!]|[\![R]\!] - P([\![R \wedge \neg B]\!]) \cdot P([\![B]\!]|[\![R]\!])$

$\mathbf{E}(\delta_{R\Rightarrow B}) = P([\![B]\!]|[\![R]\!]) + P([\![R \wedge B]\!]) - P([\![R \wedge B]\!]) \cdot [P([\![R \wedge B]\!]) + P([\![R \wedge \neg B]\!])/P([[\![R]\!])$

$\mathbf{E}(\delta_{R\Rightarrow B}) = P([\![B]\!]|[\![R]\!]) + P([\![R \wedge B]\!]) - P([\![R \wedge B]\!]) \cdot 1$

$\mathbf{E}(\delta_{R\Rightarrow B}) = P([\![B]\!]|[\![R]\!])$

random variable semantics

- let's come back on the definition of $\delta_{R \Rightarrow B}$. Crucial case: the $\neg R$-worlds where $\delta_{R \Rightarrow B}(w) = P([\![B]\!] | [\![R]\!])$.
- as Edgington (2006) puts it, the $[\![\neg R]\!]$-worlds are not divided in $R \Rightarrow B$-worlds and $\neg R \Rightarrow B$-worlds but in these worlds $R \Rightarrow B$ is so to speak "true to degree $P([\![B]\!] | [\![R]\!])$"

A	C	A→C
1	1	1
1	0	0
0	1	p(C|A)
0	0	p(C|A)

Figure: from Jeffrey (1991)

random variable semantics, cont.

- note that the "semantic value" of the conditional is uniform in the worlds where the antecedent is false. This is typically not so with counterfactuals.
- the "semantic value" of conditionals depends on the underlying partial beliefs, therefore it is an **epistemic semantics**
- if David and Paul have not the same partial beliefs, the "semantic value" of the conditionals they express may differ in the very same possible world (see below)

objections to epistemic semantics

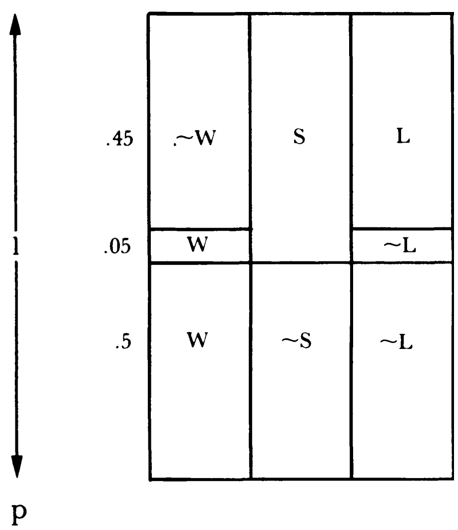
- objection 1: belief-dependency
  "...it goes against a strong intuition that we don't intend to just express our beliefs when we assert conditionals, and that we intend to say something about the way that the world is." (Bradley 2002)

an example

- objection 2: embbedings
  the main value of epistemic semantics w.r.t. Adams Logic lies in the treatment of embedded conditionals. But according to Edgington, the prediction s are bad

first example on embedding

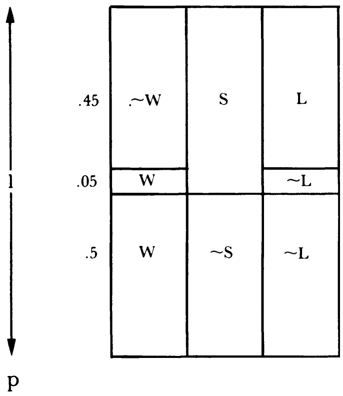
- S: David will strike the match L : the match will light W: the match is wet



- how likely "If the match is wet, if David strike it, it will light" $(W \Rightarrow (S \Rightarrow L))$?

first example on embedding

- intuitive answer: 0
- random variable semantics answer: $P(S \Rightarrow L|W)$ which can be calculated as 0.82 !
- McGee system answer: $P(W \wedge S) \Rightarrow L = 0$



second example on embedding

- another example: when A and C are incompatible, in both theories $P(A \Rightarrow B) \wedge (C \Rightarrow D) = P(A \Rightarrow B) \cdot (C \Rightarrow D)$
- let's consider a fair coin which is tossed (independently) two times

(8) It will land heads at the first toss (H_1)

(9) It will land heads at the second toss (H_2)

(10) If it lands heads at the first toss, it will land heads at the second toss ($H_1 \Rightarrow H_2$)

(11) If it does not land heads at the first toss, it will land heads at the second toss ($\neg H_1 \Rightarrow H_2$)

- intuitive probability: 1/2 (?); but the theories delivers $1/2 \cdot 1/2 = 1/4$!

partial semantics for conditionals

- second main family of proposal inside OPTION 2 = **partial semantics**
- simplest formulation of the idea (De Finetti 1937, Von Wright 1957, McDermott 1996, Milne, 1997): $A \Rightarrow C$

  (a) is true when AC,
  (b) false when $A\neg C$, and
  (c) neither true nor false when A is false
- this three-valued semantics shares some structural features with the random variable semantics: $A \Rightarrow C$ has the value of C when A is true, and an uniform value when A is not true

Triviality Results 2 Ways Out No Truth Value Summary and Perspectives

conditionals and truth-value gaps

- here are truth-tables for $\Rightarrow$ and two pairs of conjunction/disjunction ($\wedge/\vee$) and ($\cap, \cup$) (from McDermott (1996)):

ϕ	ψ	$\phi\to\psi$	$\sim\phi$	$\phi\wedge\psi$	$\phi\cap\psi$	$\phi\vee\psi$	$\phi\cup\psi$
T	T	T	F	T	T	T	T
T	F	F	F	F	F	T	T
T	X	X	F	X	T	T	T
F	T	X	T	F	F	T	T
F	F	X	T	F	F	F	F
F	X	X	T	F	F	X	F
X	T	X	X	X	T	T	T
X	F	X	X	F	F	X	F
X	X	X	X	X	X	X	X

partial semantics for conditionals, cont.

- this semantics doesn't deliver (AT) directly since $P([\![A \Rightarrow C]\!]) = P([\![A \wedge C]\!])$
- but you can see the proposition expressed by a sentence ϕ no longer as $[\![\phi]\!]$ but as a pair $([\![\phi]\!]_1, [\![\phi]\!]_0)$ where $[\![\phi]\!]_1 = \{w : v_w(\phi) = 1\}$ and $[\![\phi]\!]_0 = \{w : v_w(\phi) = 0\}$
- you can then define a notion of credence c as the probability that a sentence is true given that is has a truth-value:

$$c(\phi) = P([\![\phi]\!]_1 | [\![\phi]\!]_1 \cup [\![\phi]\!]_0)$$

- if $c(\phi) = P([\![\phi]\!]_1 | [\![\phi]\!]_1 \cup [\![\phi]\!]_0)$, then it follows immediately that

  • for any $A \in \mathcal{L}_0$, $C(A) = P(A)$
  • for any $A \Rightarrow$, $C(A \Rightarrow C) = P(C|A)$ (Adams Thesis for simple conditionals)
- the prediction of the semantics depends on the exact behavior of (ex-)Boolean connective. If one endorses the strong conjunction $\wedge$ (as apparently De Finetti 1937 did),

  • $A \Rightarrow (B \Rightarrow C) \equiv_3 (A \wedge B) \Rightarrow C$
  • $\neg(A \Rightarrow C) \equiv_3 (A \Rightarrow \neg C)$

objections

- objection 1: the strong conjunction (Edgington, Bradley)
  • $\wedge$ is appealing but a *partitioning sentence*
  $(A \Rightarrow B) \wedge (\neg A \Rightarrow C)$ is never true according to the semantics (at least one of the conditional has no truth-value) !
  • it follows that $c((A \Rightarrow B) \wedge (\neg A \Rightarrow C) = 0$!
- objection 2: the weak conjunction (Bradley 2002)
  • $\neg((A \Rightarrow B) \cap (\neg A \Rightarrow C) \equiv_T (A \Rightarrow \neg B) \cap (\neg A \Rightarrow \neg C)$

(12) It is not true that if you go left you will get to the shops and if you go right you will get to the post-office

(13) If you go left you won't get to the shops and if you go right you won't get to the post-office

objections, cont.

- last issue: we obtain Adams Thesis by equating our belief attitude towards $A \Rightarrow C$ with the credence function $c(.)$; but why should it be so ?
- Edgington (1995a) : *"The "true, false, neither" classification does not yield an interesting 3-valued logic or a promising treatment of compounds of conditionals ..."*

Bradley's semantics

- Bradley (2002) proposes a more sophisticated version of partial semantics according to which "conditionals unlike factual sentences determine propositions only in those contexts in which their antecedents are true"
- $A \Rightarrow C$ expresses the proposition $[\![AC]\!]$ in worlds where A is true and no proposition otherwise
- a sentence ϕ is true at w if the proposition expressed at w is true at w, false if the proposition expressed is false and neither if it expresses no proposition

No Truth Value

OPTION 3: No Truth Value

- let's now turn to OPTION 3 known currently as the **No Truth Value** view and held mainly by philosophers (Edgington, Bennett, Adams, Appiah, Levi)
- *"If we stick by [(AT)], we must not think of conditionals as propositions, as truth bearers...Your degree of belief that B is true, on the supposition that A is true, cannot be consistently and systematically equated to your degree of belief that something is true, simpliciter."* (Edgington 1995)
- according to this view

1. conditional assertions should not be understood as assertions of conditional propositions
2. conditional beliefs should not be understood as beliefs in conditionals

conditional assertion

- *"An affirmation of the form "if p then q" is commonly felt less as an affirmation of a conditional than as a* ***conditional affirmation*** *of the consequent. If, after we have made such an affirmation, the antecedent turns out true, then we consider ourselves committed to the consequent and are ready to acknowledge error if it proves false. Of on the other hand the antecedent turns out to have been false, our conditional affirmation is as if it had never been made."* (Quine, *Methods of Logic*)

conditional speech acts

- speech acts in general (and not only assertions) can be divided in *categorical* and *conditional* speech acts

speech act	categorical	conditional
assertion	You will go to the beach	If it's sunny, you will go to the beach
question	Will you go to the beach ?	If it's sunny, will you go to the beach ?
command	Go to the beach !	If it's sunny, go to the beach !
promise	I will go to the beach	If it is sunny, I will go to the beach

conditional attitudes

- in the same way, propositional attitudes can be divided in categorical and conditional

attitude	categorical	conditional
belief	David believes that Paul will go to the beach	David believes that if it sunny, Paul will go to the beach
desire	David desires that Paul goes to the beach	David desires that if it sunny, Paul goes to the beach
intention	David intends to go to the beach	David intends to go to the beach if it is sunny

conditional desire

- claim: a NTV theory extends more adequately to other conditional attitudes
- example: I desire that if I win the prize (W), you tell Fred straight away (F)
- propositional account: to desire that $W \Rightarrow F$ = to prefer $W \Rightarrow F$ to $\neg(W \Rightarrow F)$

  ✓ $\rightarrow$: I prefer $\neg W \vee (W \wedge F)$ to $W \wedge \neg F$!
  ✓ $>$: I prefer to be in a world whose nearest W-world is a F-world than to be in a world whose nearest W-world is a $\neg F$-world
- NTV account: to desire that $W \Rightarrow F$ = to prefer F to $\neg F$ conditionally on W = to prefer WF to $W\neg F$

Bennett's construal of NTV

- *"...the Adams theorist holds that the conditional nature of an indicative conditional comes from a relation between two of the speaker's subjective probabilities; he sees that such conditionals are not **reports** of one's subjective probabilities; so he opts for NTV, the view that in asserting $A \Rightarrow C$ a person **expresses** his high probability for C given A, without actually saying that this probability is high"* (Bennett 2003)
- analogy with so-called expressivist views for the semantics of evaluative sentences: when David says

  (14) Eating animals is wrong

  he expresses his disapproval of eating animals, he does not report that he disapproves eating animals
- why this distinction between reporting and expressing ?

dialogue involving conditional

- dialogue

(15) Z: If Pete called, he won

(16) D: Are you sure ?

(17) Z: Yes, fairly sure

- how do we understand Z's answer ?
  Z: Yes, fairly sure: I saw both hands, and Pete's was the worse
  Z: Yes, fairly sure: I calculated my ratio of subjective probabilities

dialogue involving evaluation

- dialogue

(18) Z: This film was boring

(19) D: Are you sure ?

(20) Z: Yes, fairly sure

- how do we understand Z's answer ?
  Z: "Yes, fairly sure: during the last hour, almost nothing happened."
  Z: "Yes, fairly sure: I told it during the film to my neighbor"

objections to NTV

- objection 1: "linguistic bizarreness" (Lycan): if-constructions would have no truth-conditions whereas very close constructions would have one

(21) I will leave if you leave.

(22) I will leave when you leave.

(23) If and when she submits a paper, we'll read it within a month

indicatives and subjunctives

- objection 2: parallels between indicatives and subjunctives
- most of those who endorse NTV for indicative conditionals view subjunctives as truth-valued. This makes mysterious the parallels between indicatives and subjunctives
- this is still more mysterious if one endorses the view that future indicatives are semantically similar to subjunctives (Dudman, 1983).

(24) If you dropped that vase [at *t*], your father found out [non truth-valued]

(25) If you drop that vase [said prior to *t*], your father will find out [truth-valued]

embedding

- objection 3: embedding. A truth-conditional account of conditionals provides an account of embedded conditionals (inside boolean or modal operators, conditionals, etc)
- upholders of NTV reply that

  (i) lots of embedding with conditionals are not intuitively intelligible
  (ii) truth-conditional analyses do not deal satisfactorily with compounds conditionals
  (iii) it is sufficient "to deal *ad hoc* with each kind of embedding without treating indicative conditionals as propositions" (Gibbard)

Triviality Results 2 Ways Out No Truth Value Summary and Perspectives

dealing with embeddings

- examples of *ad hoc* treatments of embeddings:

  • assert $\neg(A \Rightarrow C)$ is to assert $\neg C$ conditionally on A
  • assert $B \Rightarrow (A \Rightarrow C)$ is to assert C conditionally on AB
  • assert $(A \Rightarrow B) \wedge (C \rightarrow D)$ is to assert jointly $(A \Rightarrow B)$ and $(C \rightarrow D)$

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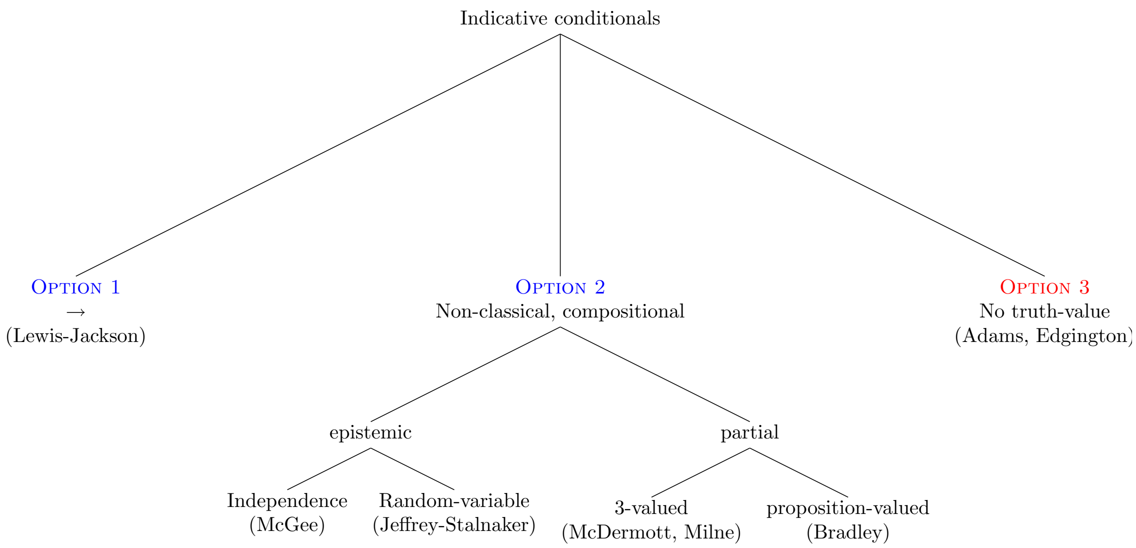
Summary and Perspectives

summary

- (1) the “Bombshell”(s): lots of triviality results showing that there is a clash between Ramsey Test and basic tenets of semantics and epistemology
- (2) the main reactions to the triviality results

summary



some inclinations and speculations

- we favor OPTION 2, but
  • (1) it is not clear that one has to make sense of arbitrary embeddings of conditionals and
  • (2) it is not clear that if the conditional has a semantics, it behaves as a binary connective (see Lecture 2)
- a view of conditionals as restrictors applied to limited kinds of embeddings ($\mathcal{L}^{\Rightarrow}_{1\frac{1}{2}}$, no conjunction or disjunction forming sentences from conditionals) could be a promising avenue
- could it survive triviality results ?

proof of Gärdenfors Triviality Result

- Lemma: (G1) and (G3) implies (I) if $\neg\phi \notin K$, then $K + \phi \subseteq K * \phi$
  Supp. $\psi \in K + \phi$.
  (1) $\phi \rightarrow \psi \in K$ (deduction theorem)
  (2) if $\neg\phi \notin K$, $\phi \rightarrow \psi \in K * \phi$ (by Preservation)
  (3) if $\neg\phi \notin K$, $\psi \in K * \phi$ (by closure and Success)
- Main Proof (by contradiction)
  Let ϕ, ψ, χ formulas pairwise incompatible but consistent with K.
  (1) by (G2) and specific assumptions, $K * \phi * (\psi \vee \chi) \neq K_{\perp}$
  (2) either $\neg\psi \notin K * \phi * (\psi \vee \chi)$ or $\neg\chi \notin K * \phi * (\psi \vee \chi)$ (by G1 and (1))
  (3) suppose w.l.o.g. that $\neg\chi \notin K * \phi * (\psi \vee \chi)$
  (4) $K + (\phi \vee \psi) \subseteq K + \phi \subseteq K * \phi$ (by Lemma)

proof of Gärdenfors, cont.

(5) $(K + (\phi \vee \psi)) * (\psi \vee \chi) \subseteq K * \phi * (\psi \vee \chi)$ (by 4 and M)
(6) $\neg\chi \notin (K + (\phi \vee \psi)) * (\psi \vee \chi)$ (by (3) and (5))
(7) $\neg(\psi \vee \chi) \notin K + (\phi \vee \psi)$ (spec.assump.+(G2))
(8) $(K + (\phi \vee \psi)) + (\psi \vee \chi) \subseteq (K + (\phi \vee \psi)) * (\psi \vee \chi)$ (by the inclusion property)
(9) $(K + (\phi \vee \psi))\ (\psi \vee \chi) = K + ((\phi \vee \psi) \vee (\psi \vee \chi)) = K + \psi$
(10) $(K + \psi \subseteq (K + (\phi \vee \psi)) * (\psi \vee \chi)$ (by (8) and (9))
(11) $\neg\chi \in K + \psi$ (by assumption and $\psi \in K + \psi$
(12) $\neg\chi \in (K + (\phi \vee \psi)) * (\psi \vee \chi)$. Contradiction

back

Independence Principle

$P(A \Rightarrow B|C)$
$= P(C \wedge (A \Rightarrow B))/P(C)$
$= P(C) \cdot P(A \Rightarrow B)/P(C)$

back

Independence Principle and Adams Thesis

$P(A \wedge C)$
$= P(A \wedge (A \Rightarrow C)$ (probabilistic MP)
$= P(A \Rightarrow C) - P(\neg A \wedge (A \Rightarrow C))$ (by laws of proba.)
$= P(A \Rightarrow C) - (P(\neg A) \cdot P(A \Rightarrow C)$ (by IP)
$= (1 - P(\neg A) \cdot P(A \Rightarrow C)$ (by laws of proba.)
$= P(A) \cdot P(A \Rightarrow C)$ (by laws of proba.)

back

an example of belief-dependency

w	P	$EVEN$	SIX	$EVEN \Rightarrow SIX$
1	1/6	0	0	1/3
2	1/6	1	0	0
3	1/6	0	0	1/3
4	1/6	1	0	0
5	1/6	0	0	1/3
6	1/6	1	1	1

belief-dependency

w	P	$EVEN$	SIX	$EVEN \Rightarrow SIX$
1	0	0	0	1/2
2	0	1	0	0
3	0	0	0	1/2
4	1/3	1	0	0
5	1/3	0	0	1/2
6	1/3	1	1	1

back

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Introduction to the Logic of Conditionals

ESSLLI 2008

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Lecture 5. Indicative and Subjunctive Conditionals

Where are we?

- Why care about the triviality results?
- They show that Adams'thesis, however plausible, cannot hold without restrictions
- They confirm that if indicative conditionals have truth conditions, then at any rate, these truth conditions are not as straightforward as those of boolean sentences.

The Indicative-Subjunctive distinction

Adams' pair

(1) If Oswald did not kill Kennedy, then someone else did.

(2) If Oswald had not killed Kennedy, someone else would have.

- Different truth-conditions
- (1) is true, given what we know about Kennedy's death. (1) is true, under the assumption that Oswald did kill Kennedy, only if one believes in conspiracy theories.

Morphology in English

- **Indicative conditionals**= IND in antecedent, IND in consequent.

(3) If Mary is rich, then she is happy.

(4) If Mary becomes rich, she will be happy.

- **Subjunctive conditionals**= SUBJ/PAST in antecedent, SUBJ/WOULD in consequent.

(5) If Mary were/was rich, she would be happy.

(6) If Mary had been rich, she would have been happy

Why “subjunctive"?

- English, present subjunctive:

(7) [The site [requires [that java scripts be enabled in your browser]]]

(8) *java scripts be enable in your website

- English, so-called past subjunctive:

(9) [Mary [wishes [she were rich]]].

(10) *Mary were rich.

- Iatridou (2000:263): *“By subjunctive I will refer to the morphological paradigm that appears in the complement of verbs of volition and/or command"*

Cross-linguistic variation

Iatridou (2000: 263):

- *“There are languages in which counterfactual morphology includes subjunctive; that is, subjunctive can be found in the complement of counterfactual wish and the antecedent of counterfactual conditionals (sometimes the consequent as well) (e.g. German, Icelandic, Spanish, Italian).*
- *Some languages do not have a subjunctive at all (Danish, Dutch).*
- *Other languages have a subjunctive but do not use it in counterfactual morphology (French, and all of the Indo-Aryan languages that have a subjunctive)."*

Salient values

(11) If (presently) Mary is rich, then she is happy/must be happy. [epistemic, present].

(12) If (50 years ago) Mary was rich, then she was happy/must have been happy. [epistemic, past]

(13) If (tomorrow) Mary becomes rich, then she will be happy. [predictive, future]

(14) If (presently) Mary were rich, she would be happy. [counterfactual, present]

(15) If (twenty year ago) Mary had been rich, she would have been happy [counterfactual, past]

Epistemic, predictive, counterfactual

Funk 1985, Kaufmann 2005

Different attitudes toward the antecedent of the conditional:

- Epistemic: subjective uncertainty about facts that are settled
- Predictive: objective uncertainty (the facts are not yet settled)
- Counterfactual: knowledge to the contrary

Remark: note that present and past indicative conditionals can very well be asserted if the antecedent has just been truthfully revealed to the speaker. In that case: the speaker indicates she takes the assumption on board (she may still doubt about it).

Anderson’s example

Anderson 1951

- Are “subjunctive" and “counterfactual" coextensional?
- Answer: No

(16) If the patient had taken arsenic, he would show exactly the same symptoms that he does in fact show.

- Indicates that it is possible that the patient took arsenic: hence non-counterfactual.

Mood and Counterfactuality

- General agreement that “subjunctive" is a misnomer (Kaufmann 2005)
- “Counterfactual" vs. “Non-counterfactual" is more adequate: semantic rather than morphological distinction
- The constraint on expressivity seems to be:

Counterfactuality $\Rightarrow$ Subjunctive

or

Indicative $\Rightarrow$ Non-counterfactuality

- Clearly, however, Subjunctive $\not\Rightarrow$ Counterfactuality.

(NB. Despite this: we may stick to old usage)

One or two kinds of conditionals?

- Dualist theories: e.g. Lewis (indicative as material, subjunctive as counterfactual)

  I cannot claim to be giving a theory of conditionals in general...there really are two different sorts of conditional; not a single conditional that can appear as indicative or counterfactual depending on the speaker's opinion about the truth of the antecedent (Lewis 1973: 3)

- Monist theories: e.g. Stalnaker. Indicative and subjunctive conditionals have identical truth-conditions, but differ in their presuppositions.

Why monism appears preferable

- A unified semantic account, to the extent that it delivers the same predictions as a dualist account, should be preferred (avoid redundancy, more explanatory)
- Lewis's theory does not work well for indicative conditionals in the first place
- Stalnaker's theory: a "Y-shaped" theory (account of semantic similarities and pragmatic differences)

Stalnaker's Y-shaped theory

Stalnaker 1975

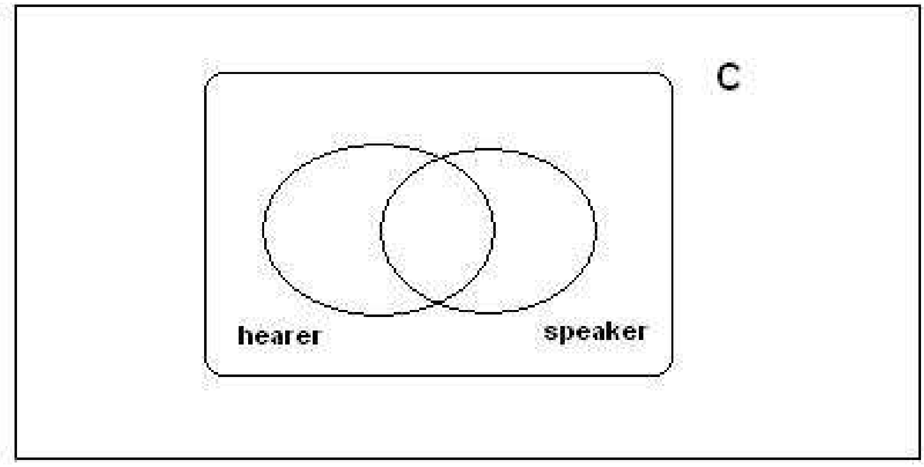
Stalnaker's aim in this paper is two-fold (killing two birds with one stone):

- Explain how indicative and subjunctive conditionals diverge from common truth-conditions
- Explain how the indicative conditional can get back some desirable properties of the material conditional, in particular disjunctive syllogism (see Lecture 1)
- In both cases: a common pragmatic mechanism

Context Representation

Context set: set of possible worlds compatible with the background information presupposed by the speaker.

- Larger than the set of belief worlds of the speaker
- Set of worlds compatible with what the speaker believes to be believed by the hearers.



Making an assumption

- *when a speaker says "if A", then everything he is presupposing to hold in the actual situation is presupposed to hold in the hypothetical situation in which A is true* (Stalnaker 1975)
- Pragmatic constraint on the selection function: *"if the conditional is evaluated at a world in the context set, then the world selected must, if possible, be within the context set as well"*

Constraint on selection

- Let C:=context set. Let $f(\phi, C)$:=$\{f(\phi, w); w \in C\}$
- Selection constraint:

$$\text{if } C \cap \phi \neq \emptyset,\ f(\phi, C) \subseteq C$$

- *"I would expect that the the pragmatic principle stated above should hold without exception for indicative conditionals" (Stalnaker 1975).*

Possibility of the antecedent

- *"indicative marking on a conditional if A, B is only felicitous relative to a world w if the context set C contains some A-world"* (Fintel, 1997)
- $C \cap A \neq \emptyset$ for "if A then B" indicative.
- Compare with probability of conditionals: $P(A > B)$ defined if $P(A) > 0$

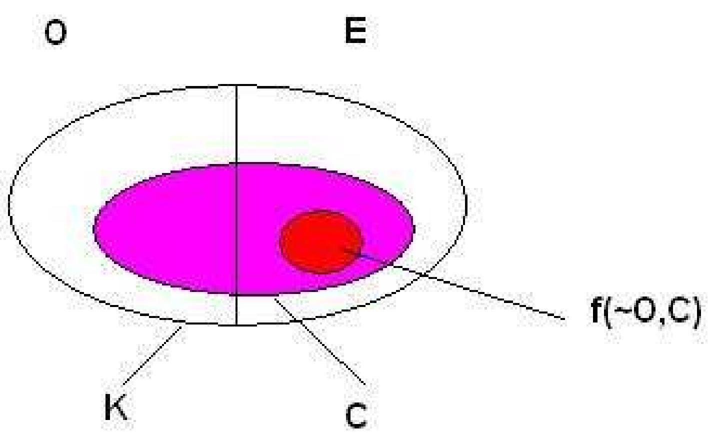
Indicative Oswald

(17) a. If Oswald did not kill Kennedy, then someone else did.
b. $\neg O > E$

K:= Kennedy was killed. Assumption: $C \cap \overline{O} \neq \emptyset$

(i) $K \equiv O \vee E$ (by definition)
(ii) $C \subseteq K$ (background knowledge)
(iii) $f(\neg O, C) \subseteq C \subseteq K$ (selection constraint)
(iv) $f(\neg O, C) \subseteq K \cap \neg O$ (cl1)
(v) hence, $f(\neg O, C) \subseteq E$. (from iv and i)
(vi) ie, $w \models \neg O > E$ (def, $w \in C$)

Remaining inside the context set



Subjunctive mood

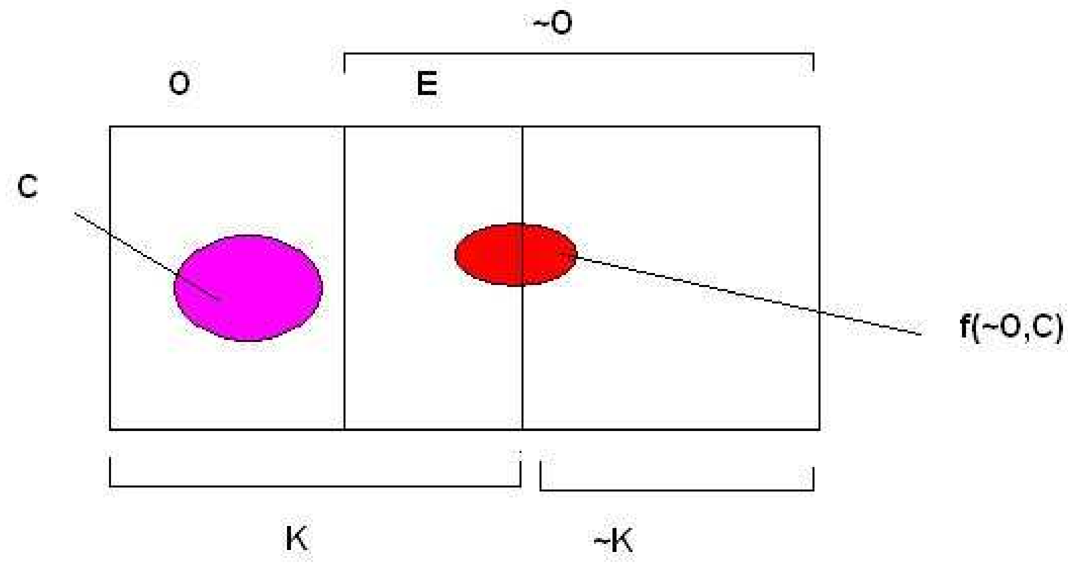
- *I take it that the subjunctive mood in English and some other languages is a conventional device for indicating that presuppositions are being suspended, which means in the case of subjunctive conditional statements that the selection function is one that may reach outside the context set* (Stalnaker 1975)
- subjunctive mood: possibly $f(\phi, C) \nsubseteq C$

Subjunctive Oswald

(18) If Oswald had not killed Kennedy, someone else would have.

- Assume $C \subseteq O$: it is assumed Oswald killed Kennedy
- Then: necessarily, $f(\neg O, C) \subseteq \neg O$, so $f(\neg O, C) \subseteq \neg C$
- Conclusion: for a counterfactual, the selection constraint is necessarily violated.
- It can be that $f(\neg O, w) \notin K$: the closest-world in which Oswald did not kill Kennedy is not a world in which Kennedy was killed.

Reaching outside the context set



Modus tollens

(19) The murderer used an ice pick. But if the butler had done it, he wouldn't have used an ice pick. So the butler did not do it.

(20) $I, (B > \neg I) \therefore \neg B$

- "the butler did not do it": cannot be a presupposition of the antecedent. Otherwise, the conclusion would be redundant.

- $C \subseteq I$
- but $f(B, w) \models \neg I$
- hence $f(B, C) \not\subseteq C$.
  Moreover:
- Suppose: $w \in B$, then $f(B, w) = w$, and $w \models \neg I$.
- So: $w \notin B$.

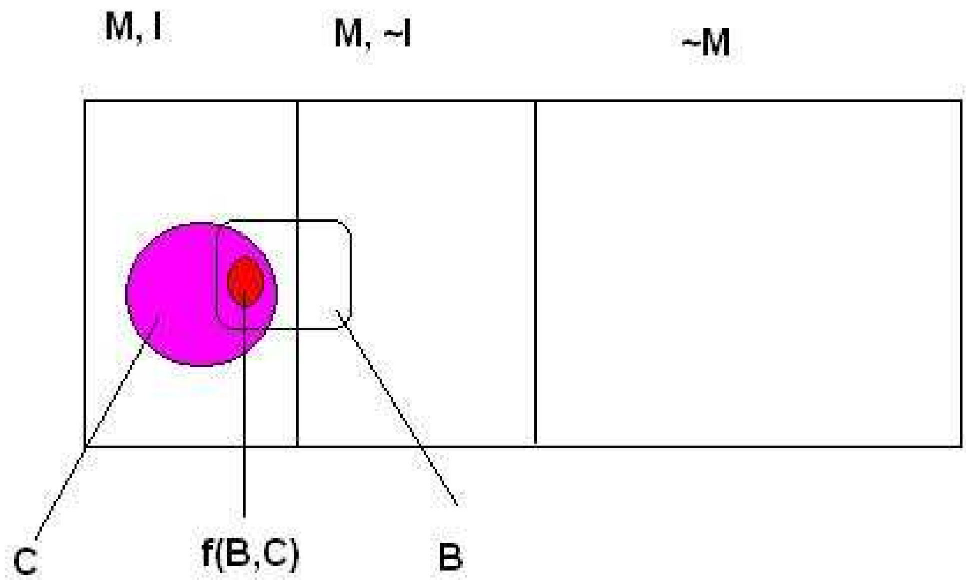
Oddities

Stalnaker's example

(21) The murderer used an ice pick. # But if the butler did it, he did not use an ice-pick. So the butler did not do it.

Origin of the oddity: "the argument is self-contradictory. the conditional presupposes that there are in fact worlds where the butler did it, there are then claimed to be worlds where no ice-pick was used, contrary to the first premise" (Fintel)

Indicative Modus Tollens



Oddities, cont.

If I did it...

Provocative oddity of O.J. Simpson's book title:

(22) "If I did it, here is how it happened"

Presupposition: maybe I did it. Yet Simpson denies his culpability. Only charitable way out: "I don't remember anything". But then: how can he tell how it actually happened!?

NB. The subjunctive version is no better for a book title, but more appropriate to prove one's innocence in court:

(23) If I had done it, here is how it would have happened.

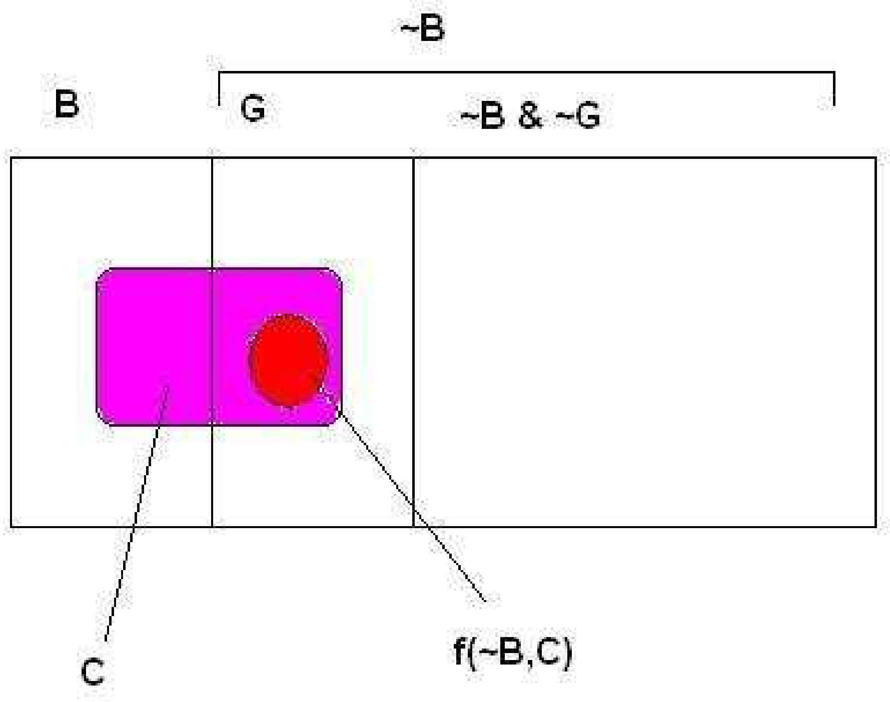
Disjunctive Syllogism

(24) a. Either the butler or the gardener did it.
b. If the butler did not do it, the gardener did it.

- Remember: $B \vee G \not\models \neg B > G$
- Stalnaker: the inference is not semantically valid, but it is pragmatically reasonable.

Appropriate disjunction

- Stalnaker's assumption: $A \vee B$ is an appropriate utterance with respect to the context set C if C allows each disjunct to be true without the other (ie for every $w \in C$, $w \models \diamond(A\neg B) \wedge \diamond(B\neg A)$):
- $C \subseteq (B \cup G)$ (after assertion)
- $C \cap B\overline{G} \neq \emptyset$, $C \cap G\overline{B} \neq \emptyset$ (Stalnaker's assumption)
- By the selection constraint: $f(\overline{B}, C) \subseteq C$
- $f(\neg B, C) \subseteq \overline{B}$ (cl 1)
- hence $f(\neg B, C) \subseteq G$



Interim summary

2 main presuppositions of indicative conditionals:

- epistemic possibility of the antecedent: $C \cap A \neq \emptyset$
- context set inclusion: the antecedent-worlds relative to the context set are part of the context set: $f(A, C) \subseteq C$
  So for indicative conditionals one can infer:
- $f(A, C) \subseteq A \cap C \neq \emptyset$

An objection by Edgington

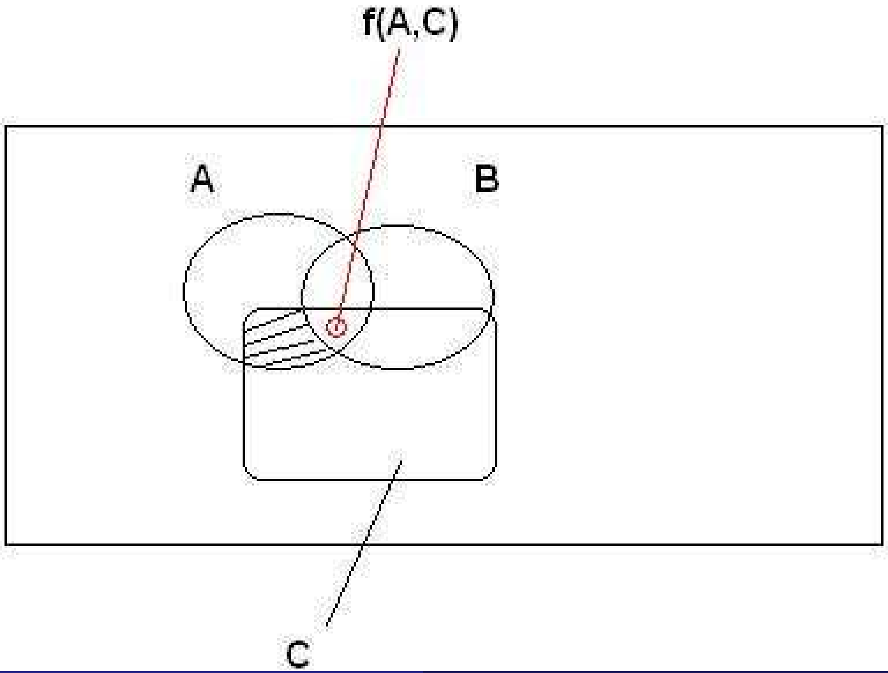
- Yesterday: we did not have the time to cover no truth value theories
- According to Edgington's version of this theory: conditionals have acceptability conditions, no truth conditions proper.
- Edgington accepts Adams' thesis
- She claims that on at least one case, the theory fares better than Stalnaker's

Edgington, cont.

- Suppose I consider both A and B possible, and am uncertain about both ($P(A) > 0$, $P(B) > 0$, $P(AB) > 0$)
- I learn that $A \wedge \neg B$ is not the case
- Then: $P(B|A) = 1$, and by Adams' thesis: I should immediately accept the conditional $A \Rightarrow B$.
- Not so for Stalnaker:
  - either $w \models A$, then $w \models B$, and $f(A, w) \models B$
  - or $w \models \neg A$. But then one can have: $f(A, w) \models B$, or $f(A, w) \models \neg B$: ie the conditional does not follow.

Stalnaker's answer

- The problem is solved assuming the selection constraint + the constraint on disjunction + $C \cap (AB) \neq \emptyset$.



Tense and Mood

Overview

- A growing literature on the topic
- Ippolito (2003), Schlenker 2005, Arregui (2006), Asher & McCready (2007), Schultz (2007),...
- Here: we shall only discuss Iatridou's theory: direct connection to Stalnaker's account.

Past morphology

- An attempt to connect verbal morphology to Stalnaker's ideas

(25) If Mary was rich, she would be happy.

(26) If Mary had been rich, she would have been happy.

- Main idea: counterfactual conditionals make a non-temporal use of past morphology

Temporal use of the Past

- Topic time: $T(t)$= the time interval we are talking about
- Utterance time: $C(t)$= the time interval of the speaker
- The Past as precedence: $T(t)$ precedence $C(t)$

(27) She walked into the room and saw a table.

Modal use of the past

- Topic worlds: $T(w)$=the worlds we are taking about
- Actual world: $C(w)$= the world(s) of the speaker
- The Past as exclusion: the topic worlds exclude the actual world [or those of the context set].
- *“the worlds of the antecedent do not include the actual world"* (Iatridou 2000)

Two values of the past

(28) If he took that syrup, he must feel better now. [temporal]

(29) If he took that syrup, he would feel better now. [modal]

(30) S'il a pris ce sirop, il doit se sentir mieux.

(31) S'il prenait ce sirop, il se sentirait mieux.

"When the temporal coordinates of an eventuality are set with respect to the utterance time, aspectual morphology is real. When the temporal coordinates of an event are not set with respect to the utterance time, morphology is always Imperfect."

Exclusion as an implicature

(32) John was in the classroom. In fact he still is.

In the same way in which counterfactuality of subjunctive conditionals can be cancelled, exclusion of the actual world/context set from the antecedent worlds can be cancelled.

Empirical adequacy

- A nice analysis of French so-called conditional mood ($\neq$ subjunctive)

(33) a. Si tu pouvais nous rendre visite, tu aimerais la ville.
b. If you could visit us, you would like the city.

- “Aimerais" = aime- + -r- + ais = ROOT + FUT + IMP
- Same pattern for all persons, singular and plural.

Veltman’s update semantics

Tichy's puzzle

Consider a man, call him Jones, who is possessed of the following dispositions as regards wearing a hat. Bad weather induces him to wear a hat. Fine weather, on the other hand, affects him neither way: on fine days he puts his hat on or leaves it on the peg, completely at random. Suppose moreover that actually the weather is bad, so Jones is wearing a hat

(34) If the weather had been fine, Jones would have been wearing his hat.

Intuitions

- Intuition: sentence false.
- Alleged prediction from Stalnaker-Lewis (acc. to Tichy): sentence should be true. In the actual world it is raining and Jones is wearing his hat. So any sunny world in which he is wearing his hat is closer than any sunny world in which he is not.

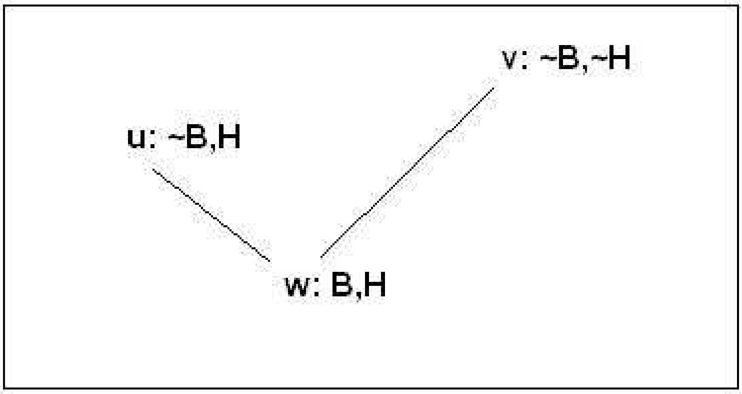
Premise semantics
Lewis 1981

- In fact the real target of Tichy's point
  Simple version of premise semantics:
- Premise set: $P(w)$ set of specific propositions true in w (remember Kratzer)
- X is A-consistent if $\cap(X \cup \{A\}) \neq \emptyset$
- X is A-maximal consistent if $\neg\exists X'$ s.t. $X \subset X'$ and X' is A-consistent
- $max_A(P(w))$:= set of maximal A-consistent sets of $P(w)$.
- Semantics: $w \models_{P(w)} A \Rightarrow C$ iff for all X in $max_A(P(w))$, $\cap(X \cup \{A\}) \subseteq C$

Illustration

Let: H= Jones is wearing a hat; B: the weather is bad.
Suppose $P(w) = \{H, B\}$.

- $\{H\}$ is the only maximal $\overline{B}$-consistent subset
- $H \cap \overline{B} \subseteq H$, ie $w \models_{P(w)} \neg B \Rightarrow H$
- Reminder: $u \leq_w v$ iff for all $X \in P(w)$ such that $w \in X$, $u \in X$.
- Let $[\![H]\!] = \{w, u\}$ and $[\![B]\!] = \{w\}$.
- Then: $w <_w u <_w v$
- Hence: $w \models \neg B \Box\!\!\rightarrow H$



Response

- When we make the counterfactual assumption that the weather is fine: no reason to maintain the fact that Jones is wearing his hat, since that depends on bad weather in the actual world.

Motivations behind Veltman's semantics

Veltman 2005

- Update semantics: the meaning of a sentence ϕ is an operation on cognitive states S: $S[\phi]$
- A modified version of simple premise semantics (cf. Kratzer)
- *"making a counterfactual assumption "if it had been the case that ϕ" in state S takes two steps. In the first step any information to the effect that ϕ is in fact false is withdrawn from S, and in the second step the result is updated with the assumption "if it had been the case that ϕ".*

States

- Language: $\phi := p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \phi \rightarrow \phi$ and $F := \phi \mid \Box\phi$.
- $\Box\phi$: "it is a law that ϕ"
- Cognitive state: $S = \langle U_S, F_S \rangle$, where either $\emptyset \neq F_S \subseteq U_S \subseteq W$ or $F_S = U_S = \emptyset$.
- F_S= facts; U_S= facts + laws

Updates

- Update for facts: $S[\phi] = \langle U_S, F_S \cap [\![\phi]\!] \rangle$ if $F_S \cap [\![\phi]\!] \neq \emptyset$, $S[\phi] = \langle \emptyset, \emptyset \rangle$ otherwise.
- Update for laws: $S[\Box\phi] = \langle U_S \cap [\![\phi]\!], F_S \cap [\![\phi]\!] \rangle$ if $F_S \cap [\![\phi]\!] \neq \emptyset$, $S[\phi] = \langle \emptyset, \emptyset \rangle$ otherwise.

Example

p= the weather is bad; q=Jones is wearing a hat

$S = W[\Box(p \rightarrow q)][p][q]$

	p	q	r
w_0	0	0	0
w_1	0	0	1
w_2	0	1	0
w_3	0	1	1
~~w_4~~	~~1~~	~~0~~	~~0~~
~~w_5~~	~~1~~	~~0~~	~~1~~
w_6	1	1	0
w_7	1	1	1

Situations

- a situation s: a partial world (partial function from atoms to truth-values)
- s forces proposition P within U_S: for all $w \in U_S$ such that $s \subseteq w$, $w \in P$.
- s is a basis for w iff s is a minimal situation such that s forces $\{w\}$ within U_S.

Example

	p	q	r
w_0	0	0	0
w_1	0	0	1
w_2	0	1	0
w_3	0	1	1
~~w_4~~	~~1~~	~~0~~	~~0~~
~~w_5~~	~~1~~	~~0~~	~~1~~
w_6	1	1	0
w_7	1	1	1

Basis for w_6: $s = \{\langle p, 1\rangle, \langle r, 0\rangle\}$
Basis for w_7: $s = \{\langle p, 1\rangle, \langle r, 1\rangle\}$

- Basis for w= minimal set of atomic facts sufficient, given the laws of nature, to select w as a unique world.

M. Cozic & P. Egré Introduction to the Logic of Conditionals ESSLLI 2008

Retraction

1. $w \downarrow P = \{s \subseteq w$; there is a basis s' for w such that s is a maximal subset of s' not forcing $P\}$.
2. $S \downarrow P = \langle U_{S\downarrow P}, F_{S\downarrow P} \rangle$ with:

   (i) $U_{S\downarrow P} = U_S$
   (ii) $F_{S\downarrow P} = \{w \in U_S$; there is $w' \in F_S$ and $s \in w' \downarrow P$ such that $s \subseteq w\}$.

Example

	p	q	r
w_0	0	0	0
w_1	0	0	1
w_2	0	1	0
w_3	0	1	1
~~w_4~~	~~1~~	~~0~~	~~0~~
~~w_5~~	~~1~~	~~0~~	~~1~~
w_6	1	1	0
w_7	1	1	1

Basis for w_6: $s = \{\langle p, 1\rangle, \langle r, 0\rangle\}$
Basis for w_7: $s = \{\langle p, 1\rangle, \langle r, 1\rangle\}$
$w_6 \downarrow [\![p]\!] = \{\langle r, 0\rangle\}$
$w_7 \downarrow [\![p]\!] = \{\langle r, 1\rangle\}$

Counterfactual assumptions

- S[if it had been the case that ϕ]=$(S \downarrow [\![\neg\phi]\!])[\phi]$
- $S \models$[if had been ϕ, would have been ψ] iff S[if had been ϕ]$\models \psi$
- Satisfaction: $S \models \psi$ iff $S[\psi] = S$.

Tichy's example

	p	q	r
w_0	0	0	0
w_1	0	0	1
w_2	0	1	0
w_3	0	1	1
~~w_4~~	~~1~~	~~0~~	~~0~~
~~w_5~~	~~1~~	~~0~~	~~1~~
w_6	1	1	0
w_7	1	1	1

$S = W[\Box(p \rightarrow q)][p][q]$
$w_6 \downarrow [\![p]\!] = \{\langle r, 0\rangle\}$
$w_7 \downarrow [\![p]\!] = \{\langle r, 1\rangle\}$
$S \downarrow [\![p]\!] = \langle U_S, U_S\rangle$, for every member of U_S extends $\{\langle r, 0\rangle\}$ and $\{\langle r, 1\rangle\}$.

Solution to Tichy's puzzle

Clearly: $(S \downarrow [\![p]\!])[\neg p][q] \neq (S \downarrow [\![p]\!])[\neg p]$

(35) It is not true that if the weather had been fine, Jones would have been wearing a hat.

Summary on Veltman

- Veltman's semantics allows us to refine the basic premise semantics
- Counterfactuals assumptions work in two steps
- Difference with indicatives: "making a counterfactual assumption does not boil down to a minimal belief revision" (Veltman 2005)

Summary for today

- Indicative implies non-counterfactuality, not the other way around
- indicative and subjunctive conditionals are most likely not so different: but different presuppositions (Stalnaker), different ways of making assumptions (Veltman)

General perspective

Problems for further exploration

- the probability of conditionals: more work needs to be done!
- Lewis-Kratzer thesis: better understanding of interaction between conditionals and modals is called for
- preserving good validities without getting the bad ones back (SDA, IE)
- tense and mood in conditionals

The End

THANK YOU!

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