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If-Clauses and Probability operators[1]

Paul Egré[2] & Mikaël Cozic[3]

Abstract

Adams' thesis is generally agreed to be linguistically compelling for simple conditionals with factual antecedent and consequent. We propose a derivation of Adams' thesis from the Lewis-Kratzer analysis of if-clauses as domain restrictors, applied to probability operators. We argue that Lewis's triviality result may be seen as a result of inexpressibility of the kind familiar in generalized quantifier theory. Some implications of the Lewis-Kratzer analysis are presented concerning the assignment of probabilities to compounds of conditionals.

Keywords

Indicative Conditionals, Probability, Triviality, Quantifier Restriction, Adams' Thesis, Lewis-Kratzer Analysis.

Résumé

On reconnaît généralement que la thèse d'Adams est convaincante du point de vue linguistique pour les conditionnels simples dont l'antécédent et le conséquent sont factuels. Nous proposons une dérivation de la thèse d'Adams à partir de l'analyse Kratzer-Lewis des clauses conditionnelles comme des restricteurs de domaine, appliquée aux opérateurs probabilistes. Nous soutenons que le résultat de trivialité de Lewis peut être vu comme un résultat d'inexprimabilité au sens de la théorie des quantificateurs généralisés. Nous présentons quelques implications de l'analyse Kratzer-Lewis pour l'assignation de probabilités à des conditionnels composés.

Mots-clés

conditionnels indicatifs, probability, trivialité, restriction de quantificateur, thèse d'Adams, analyse Kratzer-Lewis.

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1 Introduction

According to what is known as *Adams' thesis* (see Adams 1965), the probability of an indicative conditional sentence of the form $A \Rightarrow C$ should equal the conditional probability of C given A. Namely one should expect that for every propositions A and C and for every probability function p over the algebra of propositions:

(1) $p(A \Rightarrow C) = p(C|A)$

As many authors have pointed out, Adams' thesis is a *prima facie* very plausible thesis. Van Fraassen, for instance, writes: "The English statement of a conditional probability sounds exactly like that of the probability of a conditional. What is the probability that I throw a six if I throw an even number, if not the probability that: If I throw an even number, it will be a six?" (Fraassen 1976). For all its intuitive plausibility, Adams' thesis cannot hold unrestrictedly, however. As proved by Lewis (1976), Adams' thesis will hold only of a small and even trivial subclass of probability functions. Indeed, under minimal assumptions on the probabilities of A and C, Lewis proves that $p(A \Rightarrow C)$ must equal $p(C)$.

Lewis's result has been strengthened and generalized since its original publication. In recent years, in particular, Bradley has shown that another intuitive requirement on the probability of conditionals, even weaker than Adams' thesis, must lead to triviality (Bradley 2002, 2006). The requirement, which Bradley calls the *Preservation condition*, is that:

(2) a. if $p(A) > 0$ and $p(C) = 0$, then $p(A \Rightarrow C) = 0$
b. if $p(A) > 0$ and $p(C) = 1$, then $p(A \Rightarrow C) = 1$

For instance if it is possible that it is sunny tomorrow, but excluded that we will go to the beach, it seems equally excluded that we will go to the beach if it is sunny tomorrow. By a reasoning quite similar to Lewis's, however, Bradley shows that under minimal assumptions, the Preservation condition too entails that the probability of the conditional $p(A \Rightarrow C)$ must equal $p(C)$, thereby trivializing the space of probability functions for which the Preservation conditions holds.

As yet, no agreement seems to exist on the lesson to take from these triviality results. According to some, in particular Adams and Edgington (see Adams 1965, Edgington 1995), the results suggest that indicative conditionals do not have truth conditions, so that the probability of a conditional should not be interpreted as the probability of the conditional being true. Others, like Bradley, consider that conditionals do have truth conditions, but that either the algebra of propositions is to be redefined (Bradley 2002), or that the standard laws of probabilities need to be revised (Bradley 2006). Both parties to the debate however seem to agree that any satisfactory account of the probability of conditionals should preserve Adams' thesis, and indeed, few seem to be of the opinion that Adams' thesis should simply be given up (see however Kaufmann 2004 for the idea that Adams' thesis might have systematic counterexamples).

We are also of the opinion that Adams' thesis should be preserved as far as possible, but for methodological reasons that lean us to seek support for the view that while conditionals may not directly express propositions, they still make a systematic truth-conditional contribution to the meaning of sentences in which they occur. In our view, the main motivation to preserve Adams' thesis (and by way of consequence, the Preservation condition) has to do with the ordinary understanding of indicative conditionals under the scope of *overt* expressions of probability in natural language. By an *overt* expression of probability, we mean a context of the form "*the probability that ... is of* α", or "*there are* n *chances in* m *that ...*". Irrespective of the triviality results, a compositional semantics for sentences of this form is needed, and such operators do seem to express conditional probabilities when conditionals occur in their scope. For instance, as hinted from the quote by van Fraassen, in ordinary English the following two sentences are usually taken to be synonymous, and to express conditional probabilities:

(3) a. There is one chance in three that if I throw an even number, it will be a six.
  b. If I throw an even number, there is one chance in three that it will be a six.

To be sure, a sentence like (3)-b is not usually taken to mean that *whether or not I throw an even number*, the *un*conditional probability of getting a six is one in three. It can also mean that in some contexts (for instance preceded by *even*), but the important point for what concerns us is that to the extent that (3)-b expresses a conditional probability, (3)-a is interpreted to mean the same thing. The reason for this equivalence is not fortuitous, but may be seen to derive from the interaction of if-clauses with operators more generally. As argued by Lewis and Kratzer, if-clauses generally serve to restrict the domain of overt or covert operators, and if-clauses in the scope of probability operators appear to behave accordingly (see Kratzer 1991, and below).

The point we wish to make in this paper is that an adequate diagnosis of Lewis's or Bradley's triviality results and of the scope of Adams' thesis might benefit from a closer examination of the interaction of if-clauses with *overt* probability operators, in the way suggested by Lewis and Kratzer. Usually, Adams' thesis is approached from the standpoint of what we might call *covert* probability operators. For instance, it is asked what it should mean for a conditional to be asserted or believed with a particular credence (see Leitgeb 2007 for an overview). Because credences can be represented by probability distributions, the question then asked is how probabilities should be bestowed upon conditionals for arbitrary conditionals and arbitrary probability distributions. Our contention here is that one may gain insights into this problem starting bottom up, as it were, namely from a preliminary understanding of the semantics of *explicit* assignments of probabilities to *specific* conditional sentences.

The connection between the Lewis-Kratzer analysis of conditionals and Adams' thesis has been pointed out several times before us, starting with Kratzer herself (who mentions Lewis's triviality results, see Kratzer 1991: 653), and more recently by von Fintel (2006). However, we believe no attempt has been made to closely confront the triviality results themselves to the Lewis-Kratzer analysis of conditionals. In section 2, we therefore briefly review the content of the Lewis-Kratzer thesis and use it to derive Adams' thesis for simple conditionals. In that section we furthermore argue that Lewis's triviality result can be seen as additional evidence in favor of the Lewis-Kratzer analysis. In section 3, we discuss the scope of Adams' thesis regarding complex conditional sentences. If Kratzer is right that conditionals cannot be treated directly as binary connectives, but act as operator restrictors, then the assignment of probabilities to complex conditionals requires care in how the syntax of such sentences is to be spelt out. To flesh out this idea, we first examine the link between Lewis's triviality proof and conjunction of conditionals with Boolean clauses. We argue that the proof involves a step whose significance is no longer transparent when if-clauses are not seen as sentential connectives, but rather as quantifier restrictors. The upshot of our examination will be that the Lewis-Kratzer analysis provides independent motivation for the idea that conditional sentences on their own do not express propositions. Nevertheless, this does not mean that conditionals do not make a systematic truth-conditional contribution to the meaning of sentences in which they occur. In this, we exactly agree with Yalcin (2008: 1018-19) on the idea that "we can deny that indicative conditionals have possible worlds truth-conditions without denying that they have compositional semantic value".

2 The Lewis-Kratzer analysis and Adams' thesis

The so-called Lewis-Kratzer analysis of conditionals is the view that if-clauses serve to restrict the domains of various operators, and that their primary function is to perform this operation of domain restriction. In this section we briefly review Lewis's and Kratzer's arguments for this analysis and argue that from the Lewis-Kratzer thesis, we can derive Adams' thesis for simple conditionals. That is, we propose to view the thesis that the probability of conditionals is equal to the conditional probability of the consequent given the antecedent as a particular case of the generalization expressed by Lewis-Kratzer. Based on this, we argue that the Lewis-Kratzer thesis furthermore gives us a particular way of understanding Lewis's triviality result, namely as a result of inexpressibility of quantifier restriction for probabilistic operators.

2.1 If-clauses as restrictors

In his paper 'Adverbs of Quantification', Lewis pointed out that "the *if* of our restrictive if-clauses should not be regarded as a sentential connective. It has no meaning apart from the adverb it restricts" (Lewis 1975: 12). As the quote makes explicit, Lewis made no claim that this should apply to all if-clauses in natural language. However, Kratzer soon extended Lewis's remark into a systematic analysis of the contribution of if-clauses under the scope of modal operators and generalized quantifiers, and goes as far as to conclude that "the history of the conditional is the story of a syntactic mistake. There is no two-place *if...then* connective in the logical forms for natural languages. If-clauses are devices for restricting the domains of various operators. Whenever there is no explicit operator, we have to posit one." (Kratzer 1991b: 656). Indeed, for Kratzer, bare conditionals restrict a covert modality of necessity, whose interpretation varies depending on the type of necessity at issue (epistemic, deontic, and so on).[1]

The examples originally given by Lewis involve temporal adverbs such as "always, often, usually, most of the time, sometimes, ...". For instance:

(4) Always, if A then C.

(5) Sometimes, if A then C

(6) Most of the time, if A then C.

What (4) means is that all A-times are C-times; likewise, what (5) means is that some A-times are C times, and (6) means that most A-times are C times. Unlike (4), which can be paraphrased by means of the material conditional and the universal quantifier, (5) and (6) can be shown not to be truth-conditionally equivalent to sentences with a material conditional taking either wide scope or narrow scope over the respective adverb, namely to either [Sometimes]$(A \supset B)$ or $A \supset$ [Sometimes] B (and similarly for "most of the time"). Rather, in both examples, the antecedent-clause "if A" serves to restrict the domain of the adverb "sometimes" or "most of the time" (see Kratzer 1991b, von Fintel 1998).

The same point Lewis makes for temporal adverbs is briefly made by him about the "the non-connective *if* in *the probability that...if...*". According to Lewis, "It serves merely to mark an argument-place in a polyadic construction". This point is discussed in greater detail by Kratzer, who examines examples originally given by Grice to show the inadequacy of a material conditional analysis and concerning the interaction of if-clauses with expressions of probability:

(7) If Yog had white, there is a probability of 8/9 that he won.

Let us abbreviate the expression "there is a probability of 8/9 that" by the sentential operator [8/9]. As explained by Kratzer, here again neither of the material conditional analyses "$[8/9](A \supset C)$" and "$A \supset [8/9]\, C$" can deliver adequate truth-conditions for this sentence. In particular, if we only know that Yog played 100 games against Zog, won 80 out 90 games when he had white, and lost 10 out of 10 when he had black, then we can utter (7) truly

[1] We refer in priority to Kratzer's 1991a and 1991b in this paper. For a more recent and encompassing view of Kratzer's work on modals and conditionals, see the collection of papers in Kratzer (2010).

to talk about any of the 100 games that Yog played, but both of the material conditional translations would be false. Thus, what (7) says is that 8 in 9 *out of the games in which Yog had white* are games in which he won.

The analysis of Lewis and Kratzer is supposed to apply not only to temporal adverbs and expressions of probability, but to other kinds of quantifiers, irrespective of the type of the entities that are quantified over (individuals, times, events, etc.). For instance, a sentence of the form "Most letters are answered if they are less than 5 pages" appears to have the same truth conditions as "Most letters *that are less than 5 pages* are answered" (von Fintel and Iatridou 2002). In this case, the if-clause appears to play the same role as the restrictive relative clause "that are less than 5 pages". Whether this equivalence holds in full generality, namely for all quantifiers, is still a matter of debate. For instance, von Fintel and Iatridou consider the sentence (a) "Most but not all of the students who work hard will get an A" and compare it to (b) "Most but not all of the students will get an A if they work hard". One can prise apart the interpretation of these two sentences and find a model in which one is true and the other false.[2]

In what follows we will ignore these complexities, however, in order to see what the Lewis-Kratzer view implies for expressions of probability more generally. The general strategy we shall pursue is much in the spirit of the hypothesis of semantic uniformity presented in Schlenker (2006), according to which quantification over times, individuals and worlds, is constrained by general mechanisms that operate alike in the three domains. Following Lewis (1975), Schlenker points out that the sentences "most of the time, when John comes, Mary is happy", "probably, if John comes, Mary will be happy", and "most men are wise" or "most of the water is poisonous", are susceptible of essentially the same semantic analysis. On Schlenker's approach, in particular, this implies that if-clauses, when-clauses and definite descriptions obey parallel constraints.[3] In our present perspective, this implies in particular

[2]Consider a domain with four students a, b, c and d. Suppose that in case all of them work hard, only a, b, c will get an A. Suppose that only a and b actually work hard, though, and both get an A. In this case (b) seems true, but (a) false, since all of those who work hard get an A.

[3]See in particular the Appendix in Schlenker (2006) on the parallel between "most" and "probably", and Schlenker (2004) on "if" and "the". The tight link between when-clauses and if-clauses features also prominently in Lycan's (2001) account of conditionals, based on his work with M. Geis. See Lycan (2001) chapter 1, for syntactic evidence that "if...then..." constructions pattern as relative-clauses constructions. In particular, Lycan draws essentially the same generalization Kratzer makes about English: "English

that the Lewis triviality results for conditionals communicate in an essential way with undefinability results pertaining to the restriction of quantifiers, in the temporal as well as the objectual domain.

2.2 From Lewis-Kratzer to Adams

Consider a sentence such as 'sometimes, it is raining'. To represent its logical form, one option is to treat the adverb 'sometimes' as a unary operator ranging over times. A more explicit representation is to view 'sometimes' as the binary quantifier 'some' restricted by the argument 'times'. In the former case, the sentence is of form $Q_D(C)$, where Q stands for 'sometimes', D is the temporal domain of quantification, and C denotes moments where it is raining. In the latter case, the sentence is of form $Q_M(D, C)$, where Q is the binary quantifier 'some', D is its first argument or restrictor, the set of times, and its second argument or nuclear scope C denotes raining times. The difference between the two forms concerns the domain of quantification M, which can now include more objects than just times.

The Lewis-Kratzer thesis is the view that the logical form of a sentence of surface form '$Q_D C$ if A' is '$Q_{D\cap A} C$': the if-clause A restricts the domain D. Equivalently, the effect of an if-clause can be seen as that of further restricting the first argument of a sentence of form '$Q_M(D, C)$'. That is, '$Q_M(D, C)$ if A' is semantically equivalent to $Q_M(D\cap A, C)$. In what follows, we will switch back and forth between these two representations of operators as unary or binary quantifiers. For instance, consider the truth conditions of the following sentences, in which we leave implicit the domain M. Assume M to be denumerable for simplicity, and let $|A|$ denote the cardinality of the set A:

(8) a. Some D are C iff $|D \cap C| \neq 0$
 b. All D are C iff $D \subseteq C$
 c. Most D are C iff $|D \cap C| > |D \cap \overline{C}|$

The effect of using an if-clause can be seen as systematically restricting

incorporates no binary sentential connective expressed by 'if' "(2001: 91). Unlike Kratzer, however, Lycan appears skeptical about the validity of Adams' thesis and tends to dismiss the significance of the Lewis triviality results. He writes: "it would be very surprising if the semantics of natural language conditionals reflected the probability calculus in any simple way" (2001: 90). We agree, but our point is precisely that Lewis's triviality results essentially confirm the view that "if" does not behave as a binary sentential connective.

the range of the quantifier:

(9) a. Some D are C if A iff $|D \cap A \cap C| \neq 0$
b. All D are C if A iff $D \cap A \subseteq C$
c. Most D are C if A iff $|D \cap A \cap C| > |D \cap A \cap \overline{C}|$

The interesting case for our purpose concerns sentences involving *proportional quantifiers*, namely sentences such as:

(10) 80 percent of the letters are answered

(11) 1 student in 5 skips breakfast.

(12) Half of the games Yog played were easily won.

(13) Two thirds of the time, when John leaves, Mary leaves.

As happens with numerals quite generally, such sentences are ambiguous between an "at least" and an "exactly" reading (e.g. at least 80 percent of the letters vs. exactly 80 percent). For simplicity, however, we may assume that the relevant reading is always the "exactly" reading. As done above, let us symbolize by $[n/m]$ the corresponding proportional quantifier. "$[n/m]$ D are C" thus means that n in m individuals in D are C, namely:

(14) $[n/m]$ D are C iff $\frac{|D \cap C|}{|D|} = \frac{n}{m}$

For such truth conditions to hold, D must be non-empty, and the universe of discourse must be finite. For instance, a sentence like "half of the integers are even" may be judged intuitively true when considering the infinite set of all integers, but (14) would deliver inadequate truth-conditions to capture that intuition. We make both of these assumptions for the time being. The predicted effect of if-clauses on proportional quantifiers in this case becomes:

(15) $[n/m]$ D are C if A iff $\frac{|D \cap A \cap C|}{|D \cap A|} = \frac{n}{m}$

The truth conditions in (14) underlie those of probability statements of the form "there are n chances in m that C" when the domain is finite. For the latter can be analyzed as meaning that n in m eventualities of the total set D of eventualities are C-eventualities. Thus, (15) provides a derivation of the truth conditions for sentences of the form "there are n chances in m that C if A"/"there are n chances in m that if A, C". Indeed, when C and A are subsets of the domain D, (14) simplifies to $\frac{|C|}{|D|} = \frac{n}{m}$, which delivers the traditional, Laplacean definition of probability for equiprobable events in the finite case, as the ratio of favorable outcomes to the total number of outcomes. Correlatively, (15) simplifies to $\frac{|A \cap C|}{|A|} = \frac{n}{m}$, namely to the definition of the conditional probability of C given A.

Starting from the semantics of proportional quantifiers in (14), we see that a particular case of Adams' thesis (for finite state space and equiprobable events) follows from the Lewis-Kratzer analysis of the contribution of if-clauses as quantifier restrictors. In our view, the role of if-clauses as restrictors of quantifiers is fundamentally what grounds the intuition that the probability of a conditional should be equal to the conditional probability more generally.

We have seen how to derive Adams' Thesis for Laplacean probability functions. It is not so straightforward when one turns to arbitrary probability functions: what should be the general definition of $p_A(C)$, namely the probability of C when the domain is restricted by A? One way to proceed is as follows: first, consider that for every probability function p on a set W, $p(C)$ can be rewritten as $p(C) = \frac{p(W \cap C)}{p(W)}$. We could then obtain $p_A(C)$ by substituting $W \cap A$ for W, as in the case of Laplacean functions. It follows that $p_A(C) = \frac{p(W \cap A \cap C)}{p(W \cap A)} = p(C|A)$. This derivation may however seem artificial (more on this below). Another way to proceed is to rely on a framework where the relativization of probability to a domain is made explicit from the start. The Popper-Rényi axiomatization of binary probabilities as a primitive concept provides such a framework. A Popper-Rényi function $p(\cdot, \cdot)$ is a real-valued, two-place function such that for all events E_1, E_2, $E_3 \subseteq W$:

(A1) $p(\cdot, E_1)$ is a (one-place) probability function
(A2) $p(E_1, E_1) = 1$
(A3) $p(E_1 \cap E_2, E_3) = p(E_1, E_2 \cap E_3) \cdot p(E_2, E_3)$

Let $p(E)$ abbreviate $p(E, W)$, where W is the set of all possible outcomes. It follows from the axioms that if $p(E_1) > 0$, then

$$p(E_2, E_1) = p(E_1 \cap E_2)/p(E_1)$$

Let α be any real value in the interval $[0, 1]$, and let A and C stand for propositions (subsets of W). In this framework, the semantics for the probability operator $[\alpha]$, for 'the probability that ... is α' is given by:

(16) '$[\alpha]_W C$' is true iff $p(C, W) = \alpha$

From the conditionals-as-restrictors view, the clause "if A" acts as restricting W, that is:

(17) '$[\alpha]_W C$ if A' is true iff $p(C, W \cap A) = \alpha$

Suppose that $p(A) > 0$. By the preceding remark this is equivalent to:

(18) '$[\alpha]_W C$ if A' is true iff $p(C, A) = p(A \cap C)/p(A) = \alpha$

The Popper-Rényi framework is convenient because it makes explicit the relativization of simple probabilities to the domain of outcomes, and it allows us to generalize the previous derivation of Adams' thesis to the case of arbitrary probability measures. More specifically, the expression of unary probabilities as binary probabilities, combined with the axioms (A1)-(A3), guarantees that we can derive Adams' thesis from the Lewis-Kratzer operation of domain restriction by substituting $W \cap A$ for W. One could object, however, (i) that the Popper-Rényi framework makes the task too easy ; (ii) that a completely satisfactory derivation of Adams' thesis has to start from *unary* probability functions; and (iii) that the derivation for unary probabilities shown above is too artificial. Why would it be so exactly? Because we could as well have rewritten $p(C)$ as $p(C) = \frac{p(C)}{p(W)}$ and in this case $p_A(C) = \frac{p(C)}{p(W \cap A)} \neq p(C|A)$. Our first derivation does not appear to be invariant by rewriting of the probabilities. Or, to put the point slightly differently, the idea of domain restriction does not appear to apply unambiguously to probabilities. Actually, these are the reasons why we favor the Popper-Rényi framework.[4]

[4]Our clarification of this point owes a lot to a discussion with S. Kaufmann, who pointed out that conditionalization is only one among several ways of articulating domain restriction for probability operators. One informal guess about the situation could be put as follows. It seems that domain restriction implies unambiguously that $p_A(A) = 1$. As stressed notably by R. Jeffrey, $p_A(C) = p(C|A)$ iff (i) $p_A(A) = 1$ (certainty) and (ii) $p_A(C|A) = p(C|A)$ (rigidity). It is unclear whether domain restriction implies rigidity. In case it does not, the Lewis-Kratzer analysis of if-clauses implies Adams' thesis under the assumption of rigidity, and the derivation of Adams' thesis holds only in a specific domain - where rigidity holds. We leave the exploration of this point and of its implications for future work.

2.3 Triviality as undefinability

What we have argued is that Adams' thesis follows from the Lewis-Kratzer conception of if-clauses as quantifier restrictors when we consider the occurrence of if-clauses under the scope of probability operators such as 'the chances that ... are α'. In this section we propose to view Lewis's triviality result in the same way, namely as a limitative result concerning the expressibility of quantifier restriction by means of unrestricted quantification and the conditional viewed as a binary, proposition-forming connective.

Viewed in this way, the result bears a connection to similar results established for the generalized quantifier 'more than half'. Kaplan (1965) and Barwise and Cooper (1981) indeed proved that there is no way to express the binary quantifier 'more than half of the As are Bs' in terms of the unary quantifier 'more than half of all things' and the operations of first-order logic. What can be proved is a similar result, namely that assuming if-clauses to act as restrictors, there is no way to express 'the is a probability of α that C if A' in terms of the unary operator 'there is a probability of α that...' and a binary conditional connective. The result can furthermore be seen as a particular case of the Finitude result proved earlier by Hajek and presented in Hajek and Hall (1994: 91).

To state the result, we assume a denumerable set of propositional atoms $A, B, ...$, and define the following languages (without loss of generality, α may be assumed to range over rational values to keep the language denumerable):

$$\mathcal{L}_B : \quad \phi := \ A \mid \neg\phi \mid \phi \wedge \phi$$

$$\mathcal{L}_\Rightarrow : \quad \phi := \ A \mid \neg\phi \mid \phi \wedge \phi \mid \phi \Rightarrow \phi$$

$$\mathcal{L}_{P_1} : \quad \phi := \ \psi \mid [\alpha](\psi) \text{ for } \psi \text{ in } \mathcal{L}_B,\ \alpha \in [0,1].$$

$$\mathcal{L}_{P_2} : \quad \phi := \ \psi \mid \ [\alpha](\psi) \mid [\alpha](\psi, \psi) \text{ for } \psi \text{ in } \mathcal{L}_B,\ \alpha \in [0,1].$$

$$\mathcal{L}_{P_1,\Rightarrow} : \quad \phi := [\alpha](\psi) \text{ for } \psi \text{ in } \mathcal{L}_\Rightarrow,\ \alpha \in [0,1].$$

$\mathcal{L}_B$ is the language of propositional logic, $\mathcal{L}_\Rightarrow$ is the same language augmented with a binary conditional connective. $\mathcal{L}_{P_1}$ is the extension of $\mathcal{L}_B$ with unary probability operators taking only propositional formulae as arguments, and $\mathcal{L}_{P_2}$ the extension of $\mathcal{L}_{P_1}$ with conditional probability operators. $\mathcal{L}_{P_1,\Rightarrow}$

finally is the extension of $\mathcal{L}_{\Rightarrow}$ with unary probability operators. $[\alpha](\phi)$ is to be read as 'there is a probability of α that ϕ', and $[\alpha](\phi, \psi)$ as 'there is a probability of α that ψ if ϕ'.

Formulae will be interpreted over models of the form $M = \langle W, p, V \rangle$, with W a non-empty set of worlds, p a probability distribution on W, and V a valuation function for atomic sentences of $\mathcal{L}_B$. Given a formula ϕ of any of these languages, we call $[\![\phi]\!]$ the set of worlds in which ϕ is true. Given a model M, satisfaction for formulae of $\mathcal{L}_B$, $\mathcal{L}_{P_1}$ and $\mathcal{L}_{P_2}$ is as follows:

- $M, w \models A$ iff $w \in V(A)$
- $M, w \models \neg\phi$ iff not $M, w \models \phi$
- $M, w \models \phi \wedge \psi$ iff $M, w \models \phi$ and $M, w \models \psi$
- $M, w \models [\alpha](\phi)$ iff $p([\![\phi]\!]) = \alpha$
- $M, w \models [\alpha](\phi, \psi)$ iff $p([\![\psi]\!] | [\![\phi]\!]) = \alpha$

Assuming this, one can prove that there is no binary connective $\Rightarrow$ that can be used to extend $\mathcal{L}_B$, such that for every model M and every $\mathcal{L}_B$-formulae ϕ and ψ:

$$M, w \models [\alpha](\phi, \psi) \text{ iff } M, w \models [\alpha](\phi \Rightarrow \psi),$$

satisfying the following constraints:

- For all $\phi \in \mathcal{L}_{\Rightarrow}$, $[\![\phi]\!]$ is totally defined, that is for all $M, w : M, w \models \phi$ or $M, w \not\models \phi$.
- For all $\phi \in \mathcal{L}_{\Rightarrow}$, $M, w \models [\alpha](\phi)$ iff $p([\![\phi]\!]) = \alpha$.

The proof is straightforward. Let $W = \{a, b, c\}$, with $[\![A]\!] = \{a, b\}$, $[\![C]\!] = \{a\}$, and let p be the distribution that makes all worlds equiprobable. Clearly, $M \models [1/2](A, C)$. In this model, however, $[\![A \Rightarrow C]\!]$ can denote one of at most 8 distinct propositions over the Boolean algebra of W (the empty set, the singletons, the doubletons, or the whole set), but none of these propositions P is such that $p(P) = 1/2$, since each of these propositions has probability either equal to 0, 1/3, 2/3, or 1.

The connection of this result with Lewis's triviality result should be clear. It says that fixing a probability distribution, the conditional probability of C

given A cannot be equated with the probability of any conditional proposition. From a semantic point of view, the result can be interpreted as showing that the binary probability operator $[\alpha](\cdot,\cdot)$ is more expressive than the unary probability operator $[\alpha](\cdot)$ in combination with Boolean operations (i.e. $\mathcal{L}_{P_2}$ is more expressive than $\mathcal{L}_{P_1}$). The result thus shows a positive side to Lewis's original triviality result, since it establishes that restriction of unary probability operators by if-clauses adds expressiveness to a language with unrestricted probability operators taking scope over more complex formulae.

A noteworthy aspect of the result is that we do not state any explicit semantics for the conditional connective $\Rightarrow$, but only state general constraints on the satisfaction of formulae of $\mathcal{L}_{\Rightarrow}$ and $\mathcal{L}_{P_1,\Rightarrow}$. The second constraint is a constraint that says that probability operators should have a uniform semantics for formulae with and without the conditional connective. The first constraint is a constraint of bivalence, which corresponds to the idea that conditionals should systematically express propositions. This particular constraint is interesting, since it is often seen as the ingredient that should be given up for conditionals. Indeed, what the above proof shows is that in saying 'there is one chance in two that if A then C', the conditional 'if A then C' does not express any self-standing proposition. A different way to cast this observation is to go in the direction of Kratzer's analysis, namely to argue that the word 'if' does not act *directly* as a proposition-forming operator.[5] However, this remains compatible with the idea that if-clauses are devices of quantifier restriction. In the scope of an operator, if-clauses do have a systematic truth-conditional contribution to the whole sentence.

To summarize, the point of this section is that the Lewis-Kratzer thesis provides a semantic derivation of Adams' thesis regarding the probability of simple conditionals, and moreover that Lewis's triviality results can be used as further evidence in favor of the Lewis-Kratzer analysis of if-clauses as restrictors for various operators. As emphasized above, the connection we see between triviality and undefinability is very much in the spirit of similar undefinability results in generalized quantifier theory (see Peters and Westerstahl 2006). A point worth adding concerns the analogy between the generalized quantifier 'most' and the operator 'probably'. Barwise and Cooper prove that 'most As are Bs', understood as 'more than half of the As are Bs' is not definable in terms of unary 'most' and first-order operations. In general, 'probably A' may be similarly understood as: 'the probability that

[5]We are indebted to D. Rothschild for this nice way of putting it.

A is over a half', or indeed as 'the probability that A is greater than α for a context-dependent parameter α. Thus, if $[\geq \alpha]\psi$ means that 'the probability that ψ is greater than α', and $[\geq \alpha](\phi, \psi)$ 'the conditional probability that ψ if ϕ is greater than α', Barwise and Cooper's undefinability proof for 'most' can be adapted to show that $[\geq \alpha](\phi, \psi)$ is not definable in terms of $[\geq \alpha]$ and the operations of propositional logic.

3 Compounds of conditionals

Our account of Adams' thesis so far can appear to be limited in two important respects. First of all, we focused on the role of if-clauses under the scope of explicit probability operators. But what can we say about the assignment of probabilities to bare conditionals, for which no such operator is overtly expressed? Secondly, in the last section, we did not consider the semantics of complex conditionals nor of compounds of conditionals in the scope of probability operators. We considered only expressions of the form $[\alpha](A, C)$ in which A and C themselves do not contain conditional clauses. In the light of the Lewis-Kratzer analysis, what can we say about compounds of conditionals?

To answer these questions, we first make a closer examination of Lewis's triviality proof. The point of this examination is to show that Lewis's triviality result involves the assignment of probability to a conjunctive sentence involving a conditional clause and a Boolean clause. We shall argue that such conjunctive sentences do not have a clear truth-conditional contribution under the scope of overt probability operators. In the next subsection, we go on to give a broader discussion of bare conditionals and compounds of conditionals in relation to the Lewis-Kratzer analysis.

3.1 Triviality and compounds

Let us start by considering Lewis's first triviality proof. Lewis supposes that $\Rightarrow$ is a binary operator, and supposes that Adams' thesis holds for any probability function p defined for sentences of the language of propositional logic augmented with $\Rightarrow$. An important assumption concerning Boolean compounds is the factorization hypothesis, namely:

(19) if $p(A \wedge B) > 0$, $p(A \Rightarrow C|B) = p(C|A \wedge B)$

Lewis's result states that if $p(A \wedge C) > 0, p(A \wedge \neg C) > 0$, then $p(A \Rightarrow C) = p(C)$. The proof goes as follows:

(1)	$p(A \Rightarrow C) = p(C\|A)$	by Adams' thesis
(2)	$p(A \Rightarrow C\|C) = p(C\|A \wedge C)$	by Factorization
(3)	$p(C\|A \wedge C) = 1$	by the laws of probability
(4)	$p(A \Rightarrow C\|\neg C) = p(C\|A \wedge \neg C)$	by Factorization
(5)	$p(C\|A \wedge \neg C) = 0$	by the laws of probability
(6)	$p(A \Rightarrow C) = p(A \Rightarrow C\|C) \cdot p(C) + p(A \Rightarrow C\|\neg C) \cdot p(\neg C)$	Expansion by cases
(7)	$p(C\|A) = 1 \cdot p(C) + 0 \cdot p(\neg C) = p(C)$	from (1)-(6)

As Lewis notes, the result is absurd, for supposing I throw a fair die, it predicts that the conditional probability that if I get an even number, it will be a six, which is 1/3, is simply equal to the unconditional probability that it will be a six, namely 1/6. Where did things go wrong?

Our contention will be that step (6) in the proof, namely the use of Expansion by cases, is the less obvious step in the proof. Our reasons to make this claim originate from the consideration of a generalization of Lewis's triviality proof by Bradley, which Bradley (2006) presents in the form of a puzzle. Bradley's presentation is interesting for our purpose, for it involves conditional sentences where if-clauses restrict overt probability operators.

Bradley's scenario is the following: Lord Russell has been murdered. The police is certain that there was only one murderer, and that it is either the gardener, the butler, or the cook. The gardener is the less likely candidate; the cook is more likely to have done it, but less likely than the butler. Besides, their evidence can be summarized as follows:

(20) a. It is probable that it wasn't the cook.
b. It is probable that if it wasn't the butler, then it was the cook.
c. It is improbable that if it wasn't the butler, then it was the gardener.
d. It is certain, supposing that it wasn't the cook, that if it wasn't the butler, then it was the gardener.
e. It is impossible, supposing that it was the cook, that if it wasn't the butler, then it was the gardener.

To make Bradley's puzzle explicit, we will suppose that the police's credences can be expressed more explicitly as follows:

(21) a. There are 2 chances in 3 that it wasn't the cook.

b. There are 2 chances in 3 that if it wasn't the butler, then it was the cook.
c. There is 1 chance in 3 that if it wasn't the butler, then it was the gardener.
d. There is a 100 percent chance, supposing that it wasn't the cook, that if it wasn't the butler, then it was the gardener.
e. There is a 0 percent chance, supposing that it was the cook, that if it wasn't the butler, then it was the gardener.

If the police's credences are consistent, and if the corresponding conditionals express self-standing propositions, then there should be a distribution of probability p for the police's belief, such that:

(22) a. $p(\neg C) = 2/3$
b. $p(\neg B \Rightarrow C) = 2/3$
c. $p(\neg B \Rightarrow G) = 1/3$
d. $p(\neg B \Rightarrow G|\neg C) = 1$
e. $p(\neg B \Rightarrow G|C) = 0$

What Bradley observes is that "If this correctly represents the situation, then it appears that the usual laws of probability fail". Indeed, if conditionals express propositions we should have that:

(23) $p(\neg B \Rightarrow G) = p((\neg B \Rightarrow G) \wedge C) + p((\neg B \Rightarrow G) \wedge \neg C)$

which is probabilistically equivalent to:

(24) $p(\neg B \Rightarrow G) = p(\neg B \Rightarrow G|C)p(C) + p(\neg B \Rightarrow G|\neg C)p(\neg C)$

hence:

(25) $p(\neg B \Rightarrow G) = 0 \cdot 1/3 + 1 \cdot 2/3 = 2/3$

However, what was initially assumed was that $p(\neg B \Rightarrow G) = 1/3$.

What should we conclude from this example? We believe that all sentences appearing in (21), which involve overt probabilistic operators, are jointly satisfiable under a Lewis-Kratzer style semantics. However, we consider that the rule of expansion by case is no longer transparent once we refer it to the same semantics. Does it mean we should modify the laws of the probability calculus? We do not believe so. Rather, we consider that this particular step evidences the fact that conditional sentences do not di-

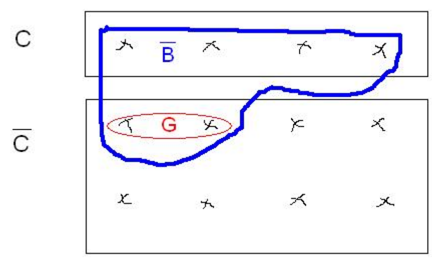

Figure 1: Bradley's scenario

rectly express propositions, in agreement with the idea that if-clauses do not contribute a meaning independently of the presence of an overt or covert operator

To see this, let us consider a finite universe W consisting of 12 worlds, with the propositions B, C and G as depicted in Figure 1, and a distribution of probability that makes all words equiprobable. Our point is that all of the sentences that appear in (21) can be translated in $\mathcal{L}_{P_2}$. For that, we need to assume that sentences of the form 'there are α chances that, given A, if B then C' can be translated as $[\alpha](A \wedge B, C)$. Such a translation appears quite natural if A and B successively act as domain restrictors for the operator.

In this model, it holds that $[1/2]B$, $[1/3]C$, $[1/6]G$, in agreement with the ordering of likelihoods assumed in Bradley's scenario, and moreover $[1]((B \wedge \neg C \wedge \neg G) \vee (\neg B \wedge C \wedge \neg G) \vee (\neg B \wedge \neg C \wedge G))$, namely it is certain that exactly one of the suspects is the murderer. Furthermore, one can check that the following sentences are all true at every point in the model:

(26)
a. $[2/3]\neg C$
b. $[2/3](\neg B, C)$
c. $[1/3](\neg B, G)$
d. $[1](\neg C \wedge \neg B, G)$
e. $[0](C \wedge \neg B, G)$

What does such a semantics tell us, then, about the step performed in (24), namely about using expansion by cases? The question may be rephrased in the following way: suppose that the above model adequately describes the way the police picture to themselves the B, C and G possibilities, their relations and likelihood. Why wouldn't the police reason according to (24) and conclude that "there are 2 chances in 3 that if it was not the butler, then it was the gardener" after all?

Our answer to this is that the police may indeed perform a reasoning by cases. However, the point we want to make is that although the probabilistic rule is by itself sound, it is not adequately mirrored in the syntax and semantics of if-clauses. In fact, a reasoning by case can be performed that essentially agrees with Adams' thesis, and that bypasses the need to handle the conjunction of a conditional clause with a boolean clause. Suppose the police wondered: "what are the chances that if it was not the butler who did it, if was the gardener?". The police might reason by case as follows:

> If it was not the butler who did it, then either it was the cook, or it wasn't the cook. So we need to determine the following. *If it was not the butler who did it*, how likely is it that it was the cook, and how likely is it that it was not the cook? For each of these cases respectively, *given that it was not the butler*, how likely is it that if it was the cook, it was the gardener, and how likely is it that if it was not the cook, it was the gardener?

This reasoning is adequately captured by the following correct rule of probability:

(27) $$p(G|\neg B) = p(G|C \wedge \neg B)p(C|\neg B) + p(G|\neg C \wedge \neg B)p(\neg C|\neg B)$$

Note that this rule is quite similar to the rule one would obtain from (24) using factorization, and applying Adams' thesis, namely:

(28) $$p(G|\neg B) = p(G|C \wedge \neg B)p(C) + p(G|\neg C \wedge \neg B)p(\neg C)$$

The latter rule, unlike the former, is not probabilistically sound, however, because it uses the base probabilities of C and $\neg C$ instead of the conditional probabilities relative to $\neg B$. However, if the police reason according to (27), based on the model we described, it still holds that $p(G|\neg B) = 1/3$. Note that the same would apply to Lewis's triviality proof. If in step (6) $p(C)$ and $p(\neg C)$ were replaced by $p(C|A)$ and $p(\neg C|A)$, then the last line of the proof would no longer be a contradiction.[6]

What about (24) in this case? Underlying (24) is the equation that $p(D) = p(D \wedge C) + p(D \wedge \neg C)$, where D and C both express possible world propositions. Consequently, when D is taken to be a proposition expressed by a conditional sentence, like $\neg B \Rightarrow G$, the rule asks us to compute the probability of the conjunction of a conditional with a boolean sentence. Our claim is that if indeed if-clauses serve to restrict the scope of operators, then it is unclear what their role should be when the if-clause appears in conjunction with further material under the scope of an overt probabilistic operator.

To be sure, consider a sentence like:

(29) It is likely that Peter will visit and Mary will visit if Peter visits.

In our view, the most natural reading of the sentence is: it is likely that Peter will visit, and *it is likely that* Mary will visit if Peter visits. In particular, the sentence does not necessarily imply that it is likely that Mary will visit. To be sure, consider the corresponding question:

(30) How likely is it that Peter will visit and Mary will visit if Peter does?

An appropriate answer would be: there is 1 chance in 2 that Peter will visit, and there is 1 chance in 3 that Mary will visit if Peter does. In particular, the question does not seem to request how likely it is that Mary will visit.

Our point can be cast differently. Suppose that A, B and C are Boolean sentences, and suppose we allowed for the word "if" to be a sentential connective, and stipulated that $[\alpha](A \Rightarrow B)$ is shorthand for the restricted operator $[\alpha](A, B)$, whose truth-conditions are transparent. Our intuition is that it is

[6]Interestingly, Kaufmann (2004) has put forward the idea that a rule such as (28), which he calls *local* conditional probability, might be a rational rule of belief update, and for that matter that it could provide a counterexample to Adams' thesis. We agree with Kaufmann that it may be common practice to reason by cases according to (28), however we believe it is a reasoning fallacy, close to the base rate fallacy, which should not bear on the validity of Adams' thesis. In our view, a theory of reasoning according to local probability is certainly relevant for the psychology of conditional probability, but we are skeptical that it should be part and parcel of the semantics of conditional probability. In that, we share the skepticism expressed by Douven (2008) in his criticism of the normative character of Kaufmann's rule. To borrow a distinction nicely made by S. Kaufmann in conversation, we agree that an account of local probability is certainly relevant for a performance theory of conditionals and probability, but we doubt that it should be part of the corresponding competence theory.

simply unclear what a sentence like $[\alpha]((A \Rightarrow B) \wedge C)$ will be shorthand for, and what meaning it should have. If the role of the if-clause A is to restrict the domain of an operator, then here it appears that A cannot restrict the domain of the probability operator $[\alpha]$, since the conditional clause in this case is syntactically dominated by the conjunction. What our examination suggests therefore is that it may be misguided to ask for the probability of arbitrary compounds of conditionals, as soon as it is unclear what operator if-clauses restrict in such compounds.

3.2 Further remarks on compound conditionals

Our approach may appear too limitative however with regard to compounds of conditionals. Like Adams, we can account for the probability of simple conditionals, which do not contain embedded conditionals in the antecedent and consequent clause. But Adams' logic is often felt too restrictive with regard to the expressiveness of natural language. As McGee writes, Adams' logic "does a marvelous job of accounting for how we use simple conditionals", but "it tells us nothing about compound conditionals or about Boolean combinations of conditionals" (McGee 1989: 485). We are in the same position so far.[7] What we need to examine in this section is whether it makes sense to ask for the probability of arbitrary compounds of conditionals. The position we are inclined to defend is that it makes sense to ask what the probability of a given sentence is if the answer can be expressed as a conjunction of sentences of the form: 'there is a probability of α that (if ϕ,) ψ', where ϕ and ψ are sentences that express propositions. This does not mean for us that ϕ and ψ need be purely Boolean formulae. Such formulae could be built out of operators themselves. What matters, however, is that they be truth-conditionally evaluable.

[7]In Adams logic, the conditional is treated as a binary sentential connective, but this connective always takes highest priority over Boolean formulae, and cannot embed other conditional formulae. Because of that, a formula $(A \Rightarrow B)$ in Adams' logic may be read as equivalent to a formula of the form $[Probably](A, B)$ in the kind of languages with explicit probability operators that we introduced, and a Boolean formula ϕ as a formula of the form $[Probably](\phi)$. In Adams' logic, ψ is a consequence of ϕ if for every probability distribution p, $p(\phi) \leq p(\psi)$. We could imagine to define an analogous notion of consequence between formulae prefixed by $[Probably]$, namely to impose that $[Probably]\psi$ is a consequence of $[Probably]\phi$ iff every model (W, p, V) that makes any formula $[\alpha]\phi$ true makes $[\alpha]\psi$ true. However we are not interested in logical consequence in this paper. Our focus is primarily on the logical form of conditional sentences.

3.2.1 Bare conditionals

Suppose Mary utters the indicative conditional: 'if I toss this coin, it will land heads'. What do we ask when we wonder how likely it is that what Mary said will be true? According to Kratzer, a bare indicative conditional such as 'if I toss this coin, it will land heads' is a covert modalized sentence of the form $[Must]$(*Toss*, *Heads*), in which the if-clause restricts the necessity operator 'Must'. The point of Kratzer's analysis is that bare conditionals can express different propositions depending on the assumptions that are made on the interpretation of 'Must'. 'Must' can have varying modal force, in particular, and depending on the assumptions on what Kratzer calls the modal base and the ordering source, the truth conditions for restricted 'Must' can yield various conditionals, including the material conditional, or the strict conditional (see Kratzer 1991a: 649).

For instance, Mary may be expressing a strict conditional, that 'if I toss this coin, it will necessarily land heads'. If I believe the coin to be fair, I may end up giving probability 0, rather than 1/2, to Mary's statement, if what I evaluate is the probability of the strict conditional proper. But I may assign the conditional probability 1/2, if I simply wonder what the chances are that the coin will land heads. Thus it is possible to have: $[1/2](Toss, Heads)$ and $[0]([Must](Toss, Heads))$, if no world supports the strict conditional interpretation of $[Must]$.

Suppose however that we are dealing with a special lottery, such that you will be assigned one of three coins with equal probability. Two of them are biased and will invariably land heads. The third one is a fair coin. We hear a contrast between:

(31) There are 2 chances in 3 that if I toss this coin, it will land heads.

(32) There are 2 chances in 3 that necessarily if I toss this coin, it will land heads.

(32) strikes us as true in this scenario. (31) can be judged false, however, because the conditional probability of the coin landing heads, given that I toss it, is 5/6 in this case. Kratzer's theory is helpful here, since it can account for these contrasts. More generally, this suggests that when evaluating the probability of various conditional sentences, we have to consider carefully what their logical form is.[8]

[8] We are indebted to B. Spector for this example and for the judgment. Spector came up

3.2.2 Conjunctions of conditionals

Let us now turn to compounds of conditionals proper. There appear to be natural sentences involving conjunctions of conditionals. One case which we already mentioned concerns one-sided conjunctions, for instance in:

(33) Peter will leave, and Mary will leave if Peter leaves.

In most theories of indicative conditionals, this sentence is equivalent to 'Peter will leave and Mary will leave'. Because of that, one may expect the probability of 'Peter will leave and Mary will leave if Peter leaves' to be identical to the probability of 'Peter will leave and Mary too'. Suppose however that the probability of Peter leaving is 2/3, and the conditional probability of Mary leaving based on Peter leaving is also 2/3. In this case, the probability of 'Peter will leave and Mary too' is 4/9. As hinted above, it seems to us that in this case, it would be natural to assert: 'there are 2 chances in 3 that Peter will leave and (that) Mary will leave if Peter leaves'. Or even, 'it is more likely than not that Peter will leave and (that) Mary will leave if Peter leaves'. But it would be false to say 'it is more likely than not that Peter will leave and Mary too'. Conversely, suppose that the probability of Peter leaving is 99/100, and that the conditional probability of Mary leaving if Peter leaves is 3/100. Could we say in that case: 'the probability is more than 1/2 that Peter will leave and (that) Mary will leave if Peter leaves'? This strikes us as unnatural, given that the conditional probability of Mary leaving if Peter leaves is much below a half.

Because of that, we believe that there is a tendency to interpret sentences of the form 'the probability is α that B and if A then C' distributively, namely as: 'the probability is α that B and the probability is α that if A then C'. Does it mean, however, that it is impossible to understand 'there is a probability of α that A and if A then B' as 'there is a probability of α that A and B'? If we follow Kratzer's thesis, this may still be possible if the form of this sentence is: $[\alpha](A \wedge [Must](A, B))$. The sentence will be equivalent to $[\alpha](A \wedge B)$ if the sentence $(A \wedge [Must](A, B))$ can be shown to express the same proposition as $(A \wedge B)$. For instance, if $[Must](A, B)$ is interpreted as

with the example in response to an objection made by D. Rothschild (2009) to a particular prediction of Kratzer's account, namely the prediction that 'there are n/m chances that p' and 'there are n/m chances that it is true that p' need not be equivalent. Spector's original judgment concerned the same pair in which 'it is true that' is used instead of 'necessarily'.

denoting the closest A-worlds that are B worlds, then under the assumption of centering, $A \wedge [Must](A, B)$ will be equivalent to $A \wedge B$.

There are many examples, similarly, of so-called two-sided conjunctions, in which a conditional clause appears on each side of the conjunction, as in:

(34) If it is an even number, it will be above three, and if it is an odd number, it will be below three.

Suppose that we are talking of a fair die. According to McGee's calculus for compound probabilities, the probability of this complex sentence is 2/9.[9] According to McDermott, however, the intuition we should have is that the probability of this conjunction of conditionals is equal to the probability that the number will be either 4, 6 or 1, namely 1/2.[10] We tend to share McDermott's intuition here. However, the issue is whether we can say: 'the probability is 1/2 that the number will be above three if even, and below three if odd', without implying that the probabilities of each conditional is 1/2 (compare, again, with 'it is likely that the number will be above three if even, and below three if odd', which seems to imply that each option is likely).

On the other hand, it is quite natural to say: 'the probability is 1/2 that the number will be even and above three, or odd and below three'. The latter can be expressed in our framework, but the question is how we can derive this equivalence from any suitable logical form for (34). Within Kratzer's framework, we could translate (34) as: $[1/2]([Must](Even, > 3) \wedge [Must](Odd, < 3))$. There is a way of recovering the equivalence then with $[1/2]((Even \wedge > 3) \vee (Odd \wedge < 3))$ if we can assume that $Must(A, B)$ in this context is truth-conditionally equivalent to the material conditional $A \supset B$. As mentioned above, Kratzer's framework makes room for this

[9] The so-called Independence Principle implies for any two-sided conjunction (McGee 1989: 500) that
$p((Even \Rightarrow > 3) \wedge (Odd \Rightarrow < 3)) = \frac{1}{p(Even \vee Odd)} \times [p(Even \wedge > 3 \wedge Odd \wedge < 3) + p(\neg Even \wedge Odd \wedge < 3) \times p(Even \Rightarrow > 3) + p(Even \wedge > 3 \wedge \neg Odd) \times p(Odd \Rightarrow < 3)]$,
which can be simplified as:
$p((Even \Rightarrow > 3) \wedge (Odd \Rightarrow < 3)) = p(Odd \wedge < 3) \times p(Even \Rightarrow > 3) + p(Even \wedge > 3) \times p(Odd \Rightarrow < 3)$,
therefore:
$p((Even \Rightarrow > 3) \wedge (Odd \Rightarrow < 3)) = 1/6 \times 2/3 + 2/6 \times 1/3 = 2/9$.

[10] This can be derived from McDermott's three-valued semantics where a conditional is neither true nor false if the antecedent is false and a conjunction is true if one of the conjuncts is true and the other neither true nor false.

possibility depending on the assumptions governing the semantics of 'Must' in the model. What our approach does not explain then, however, is why this assumption would be natural in this context, given that in other contexts we are not computing the probabilities of material conditionals.

3.2.3 Nested conditionals

Finally, some words are needed about nested conditionals. There are two kinds of nested conditionals, right-nested and left-nested. For instance:

(35) If Mary stays, then if Sue leaves, John will be sad.

(36) If Mary stays if Sue leaves, then John will be sad.

In propositional logic, (35) would be translated as $M \supset (S \supset J)$, and (36) as $(S \supset M) \supset J$. Most accounts of conditionals agree that right-nested conditionals can be interpreted in agreement with the law of import-export, namely as equivalent to $(M \wedge S \supset J)$ in this case. Our intuition for right-nested conditionals is the same. In particular, we believe that sentences of the form: 'there are n in m chances that if A, then if B then C' are syntactically analyzed as: 'there are n in m chances that if A and B, then C'. If we make this into a postulate, we can agree with the idea that right-nested conditional clauses in the scope of an operator successively restrict the domain of the operator. If this assumption is correct, it implies that the antecedents of right-nested conditionals can be simplified into Boolean conjunctions under the scope of probabilistic operators.

A more puzzling case concerns left-nested conditionals. In this case, our intuition is that left-nested conditionals also end up being interpreted as equivalent to simpler conditionals. For instance, (36) above appears to mean the same thing as: 'if Sue leaves and Mary stays, John will be sad'. Can we uphold this equivalence for all left-nested conditionals? To answer this, consider the case of so-called conditional conditionals, namely conditionals that are both left-nested and right-nested. Kaufmann (2009) gives the following examples:

(37) If Harry will pass if he takes the test, he will win if he is selected for the show.

(38) If the vase will crack if it is dropped on wood, then it will shatter if it is dropped on marble.

In the case of (37), our intuition is that the sentence means the same thing as: 'if Harry takes the test and passes it, then he will win if he is selected for the show', which we see again as equivalent to 'if Harry takes the test and passes it, and is selected for the show, then he will win'. (38) is obviously more complex, since it does not mean 'if the vase is dropped on wood and cracks and is dropped on marble,...'. However, this strikes us a case that is amenable to a Kratzerian logical form. Consider the following sentence:

(39) There are 2 chances in 3 that if the vase will crack if it is dropped on wood, then it will shatter if it is thrown on granite.

A candidate logical form is: $[2/3]((([Must](Wood, Crack) \wedge Marble), Shatter)$. The restrictor of the probabilistic operator can be assumed to express a possible world proposition in this case. Consider for instance, a model consisting of equiprobable worlds that all make true $[Must](Wood, Crack)$ (however the semantics of $[Must]$ should go here), and in which the vase is dropped on marble. Suppose that two thirds of the worlds in which the vase is thrown on marble are worlds in which the vase shatters. Then the conditional will be true, and it is evaluated according to Adams' thesis.[11] In our view, therefore, right-nested and left-nested conditionals are not essentially different. If so, we believe that both kinds of conditional clauses are simplified into simpler conditional clauses in the process of meaning computation.

4 Concluding remarks

In the first half of this paper, we have argued for two main claims. The first is that the intuition that Adams' thesis is linguistically compelling for the interpretation of the probability of simple conditionals can be justified on the basis of the Lewis-Kratzer analysis of the function of if-clauses as domain restrictors for various operators. The second main claim we put

[11] A connection may be established between the present analysis of nested conditionals and the account outlined by Gibbard (1981), in which Gibbard considers that nested conditionals can be assigned probabilities if they are equivalent to a sentence that expresses a proposition. In this, our account converges with specific intuitions of the 'no truth value' view of conditionals, but as repeated throughout the paper, we do not endorse such a view, since we see the Kratzerian analysis as a better way of articulating some of the good intuitions contained in it. See also von Fintel (2006).

forward is the idea that Lewis's triviality result can in turn be seen as further evidence for the Lewis-Kratzer thesis. In particular, we have suggested that for a simple language with probabilistic operators, triviality can be recast as a result of undefinability of the restriction of probabilistic operators by means of unrestricted operators taking scope over a proposition build out of a conditional connective.

In the second half of this paper, we have examined some predictions that the Lewis-Kratzer analysis of if-clauses gives us about the probabilities of various conditional sentences, in particular for compounds of conditionals. We have tried to suggest that in order to compute the probabilities of complex conditional sentences, it may be enough to rely on simplification rules that lead to a conditional clause with a truth-conditional evaluable antecedent and consequent. Our account needs to be refined, however, in particular in the light of conjunctions of conditional sentences, for which the equivalence with simpler Boolean clauses needs *ad hoc* assumptions. Nevertheless, a common element to all the examples we reviewed in that section is the idea that the logical form of compounds of conditionals may be quite intricate if silent operators can be postulated in the way suggested by Kratzer. In some cases, we have seen clear evidence for the semantic ambiguity of some of these conditional sentences.

To conclude, we should highlight some elements that may plead in favor of a different account. Interestingly, Lewis himself did not connect his triviality result with his remarks on adverbial quantification, although, as von Fintel (2006) points out, Lewis was aware of remarks that Belnap (1970) had made earlier on the link between restricted quantification and trivalent logic (see Lewis 1975). What Belnap suggested is that instead of considering that conditionals are not adequately expressed by a binary sentential connective, we may stick to simple logical forms for conditional sentences, but get rid of the bivalence assumption. As we emphasized in section 2.3 above, a central assumption in the version of triviality that we proved is that conditional sentences should be true of false at every world. A different strategy to account for Adams' thesis would therefore be to get rid of this assumption of bivalence, whether by endorsing trivalence or by allowing the logic to be partial. Several authors have explored this issue, in particular recently Rothschild (2009), Kaufmann (2005), Bradley (2002), and earlier by McDermott (1996), Stalnaker and Jeffrey (1994) or Jeffrey (1991). An advantage of their strategy over the present one is that they make semantic predictions regarding the probabilities of arbitrarily complex conditional sentences, without having to stipulate syntactic restrictions. In comparison, our account remains insufficiently predictive.

Philosophically, however, we find it important to emphasize the following element of convergence. From triviality, trivalent or partial semantics for conditionals draw the lesson that conditionals do not always express propositions, but they maintain that if-clauses have a systematic truth-conditional contribution to sentences in which they appear. The same holds of the Lewis-Kratzer's view of if-clauses: if-clauses do not act directly as proposition-forming operators, but this is compatible with the idea that sentences in which they appear can express such propositions. Either way, the lesson from triviality appears to be that either the syntax or the semantics of conditionals needs to be refined.

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