

Which Logic for the Radical Anti-Realist?

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(Anti-)Realisms, Logic and Metaphysics, Nancy

Anti-Realism and Logic

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realism	\Rightarrow	classical logic
moderate anti-realism	\Rightarrow	intuitionistic logic
radical anti-realism	\Rightarrow	linear logic

Semantic anti-realism

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- ▶ But realist theories of meaning cannot meet the manifestation challenge.
- ▶ Instead, one should favor a semantic in terms of assertibility conditions such that the conditions under which a statement is true should necessarily be recognizable as such
- ▶ What has this to do with logic?

Two ways of going intuitionistic

- ▶ High level revisionism: a direct argument from semantic anti-realism against bivalence and the excluded middle (EM).
- ▶ Low level revisionism: intuitionistic logic as a by-product of proof-theoretic semantics for logical constants.

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→ The disagreement with the realist bears upon the domain of validity of the excluded middle.

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- ▶ In the case of logical constants, harmony means that any detour consisting of an introduction rule followed by an elimination rule for the same connective has to be eliminable.
- ▶ *In a mono-conclusion natural deduction*, intuitionistic rules do satisfy harmony, but there is no way of adding a rule to get classical logic while preserving harmony.

Converging levels

A theory of meaning in terms of verification is bound to yield a notion of truth for which bivalence fails to hold for many sentences which we are unreflectively disposed to interpret in a realistic manner. (Dummett, 1993, p.75)

Low-level revisionism shows that a theory of meaning in terms of verification yields precisely the logic it is bound to yield on account of high-level revisionism.

Decidability in principle vs decidability in practice

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Argument: the epistemic constraint on truth has no bearing if it appeals to methods which are *in practice* out of reach.

Decidability should be replaced by *feasibility*.

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Idea: Some structural rules play a special role w.r.t. to epistemic idealization.

$$\frac{\Gamma \vdash \phi}{\Gamma, A \vdash \phi} \quad \frac{\Gamma, A, A \vdash \phi}{\Gamma, A \vdash \phi}$$

If one wants to “unidealize” logic, one has to control these structural rules.

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- ▶ It is expected that this tension will be reinforced for the stronger revisionism of radical anti-realist

Splitting of connectives

- ▶ Compare the two pairs of rules for conjunction:

$$\frac{\Gamma, A \vdash C}{\Gamma, A \wedge B \vdash C}$$

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$$\frac{\Gamma, A, B \vdash C}{\Gamma, A \wedge B \vdash C}$$

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- ▶ The two pairs are equivalent with structural rules but no longer without
⇒ two connectives: multiplicative conjunction \otimes and additive conjunction $\&$

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- ▶ $\&$ seems OK for the exploitation since $A \& B \vdash A$ but not for the conditions of assertion since $A, B \vdash A \& B$ is not derivable
- ▶ How to reconcile pre-theoretic intuitions with splitting ?

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Radical anti-realist's discomfort

- ▶ Two main options for the radical anti-realist:
 1. claim that one of the two connectives is the "true" conjunction
 2. claim that both connectives are legitimate and that there does not exist such thing as a single unifying logical constant
- ▶ It's not easy to live without structural rules !

High level radical revisionism

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To justify his choice of a substructural logic, the radical anti-realist has to provide a low level argument, and in any case, the convergence of level will be lost.

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We would like the full picture:

	moderate anti-realism	radical anti-realism
logical criterion	harmony	?
structural criterion	?	?

Harmony for the moderate anti-realist

Let us consider the following pair of rules, which can be thought of as giving assertability and exploitability conditions for some kind of conjunction:

$$\frac{\Gamma \vdash A \quad \Gamma' \vdash B}{\Gamma, \Gamma' \vdash A \wedge B} \wedge\text{-intro} \quad \frac{\Gamma \vdash A \wedge B}{\Gamma \vdash A} \wedge\text{-outro} \quad \frac{\Gamma \vdash A \wedge B}{\Gamma \vdash B} \wedge\text{-outro}$$

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These rules are ok for the moderate anti-realist, because from:

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one can get:

$$\frac{\Gamma \vdash A}{\Gamma, \Gamma' \vdash A} W$$

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Let us call this strong harmony.

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logical criterion	harmony	strong harmony
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A BHK-like proposal: **Preservation of effectivity**

A structural rule of the form
$$\frac{\Gamma \vdash \phi}{\Gamma' \vdash \phi'}$$

is admissible if, if there exists an effective means to transform justifications for all sentences of Γ into a justification for ϕ , there exists an effective means to transform justifications for all sentences of Γ' into a justification for ϕ'

Preservation of effectivity, cont.

Structural rules for intuitionistic logic satisfy the principle of preservation of effectivity.

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Let us have a look at weakening:

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Start with an effective method to get a justification for ϕ from justifications for sentences in Γ .

Do we have also an effective method to get a justification for ϕ from these justifications plus a justification for A ?

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Yes! Just drop the unnecessary justification for A .

Structural rules and radical anti-realism

We now have:

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structural criterion	preservation of effectivity	?

We shall now discuss various ways of filling the last blank.

1. Token preservation
2. Preservation of local feasibility
3. Preservation of global feasibility

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Problem: Are cognitive resources really on a par with consumption goods?

Local feasibility

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Consider weakening again.
$$\frac{\Gamma \vdash \phi}{\Gamma, A \vdash \phi}$$

Let us assume that we have an effective and *feasible* method to get a justification for ϕ from justifications for sentences in Γ . What about our method for turning a justification for ϕ from these justifications plus a justification for A ? Is it less feasible?

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No! The idea is just to do as if no new information has been given, and to stick to the good old feasible procedure, *even if* we could now try to use in non feasible ways the new justification we will be provided with.

Still...

The radical anti-realist might reply that:
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This does refute our objection to Preservation of local feasibility.

Because by saying that, the radical anti-realist makes a shift from the question of the preservation of feasibility for transformation procedures to the question of the feasibility of establishing that ϕ follows from Γ .

Global feasibility

The story would go like this:

Preservation of global feasibility

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Unfortunately, this is not true.

Logic	Complexity of \vdash
Prop. intuitionistic logic	PSPACE-complete
Linear Logic	Undecidable
MALL	PSPACE-complete

Perspective

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Any idea? Provide an alternative notion of justification invalidating structural rules.

Perspective, cont.

→ As suggested by Marion, game-theoretics semantics would do:

- ▶ Winning strategies can be identified with justifications.
- ▶ Possession of a winning strategy is not preserved by structural rules.

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In particular (Blass, 1992), it can be the case that I have a justification for

$$\vdash A, A$$

without having a justification for

$$\vdash A$$

Perspective, cont.

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- ▶ But what is the interpretation of infinitely long “justificatory debates” for the radical anti-realist?

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- ▶ But what is the interpretation of infinitely long “justificatory debates” for the radical anti-realist?

→ Question: could one base a game-theoretic semantics on a class of zero-sum two players game in which failure of determinacy would come from the players cognitive limitations (restrictions on available strategies) instead of the size of the game?