Choice and Computation

Remarks on the relevance of computability and complexity for the foundations of bounded rationality

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introduction

- computational studies assert that bounded rationality is one of their motivations (Lewis 1985, Richter and Wong, 1999)
- bounded rationalities studies mention often results of computational studies (Simon 1978)
- computational amendments of classical models (Rubinstein 1998, Neyman 1998, Papadimitriou-Yannakakis 1994)

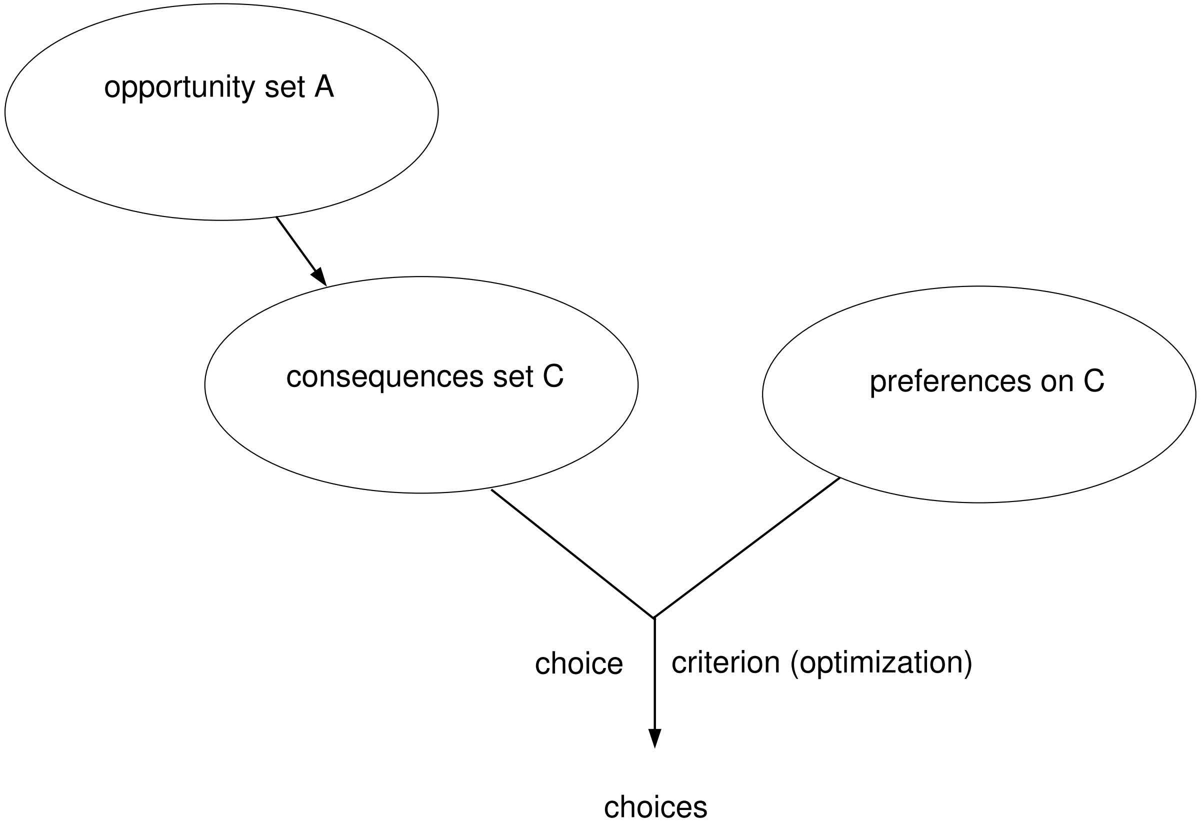
introduction, *cont.*

- Question 1 : What is the basic connexion between computational studies and bounded rationality ?
  $\hookrightarrow$ Section 1, ***The basic connexion***
- Question 2 : How can computational studies help to *appraise* choice models ?
  $\hookrightarrow$ Section 2, ***Impossibility results***
- Question 3 : How can computational studies help to *improve* choice models ?
  $\hookrightarrow$ Section 3, ***Computational constraints in repeated games***

section 1

The basic connexion

sec.1, elementary choice model (ECM)



opportunity set A

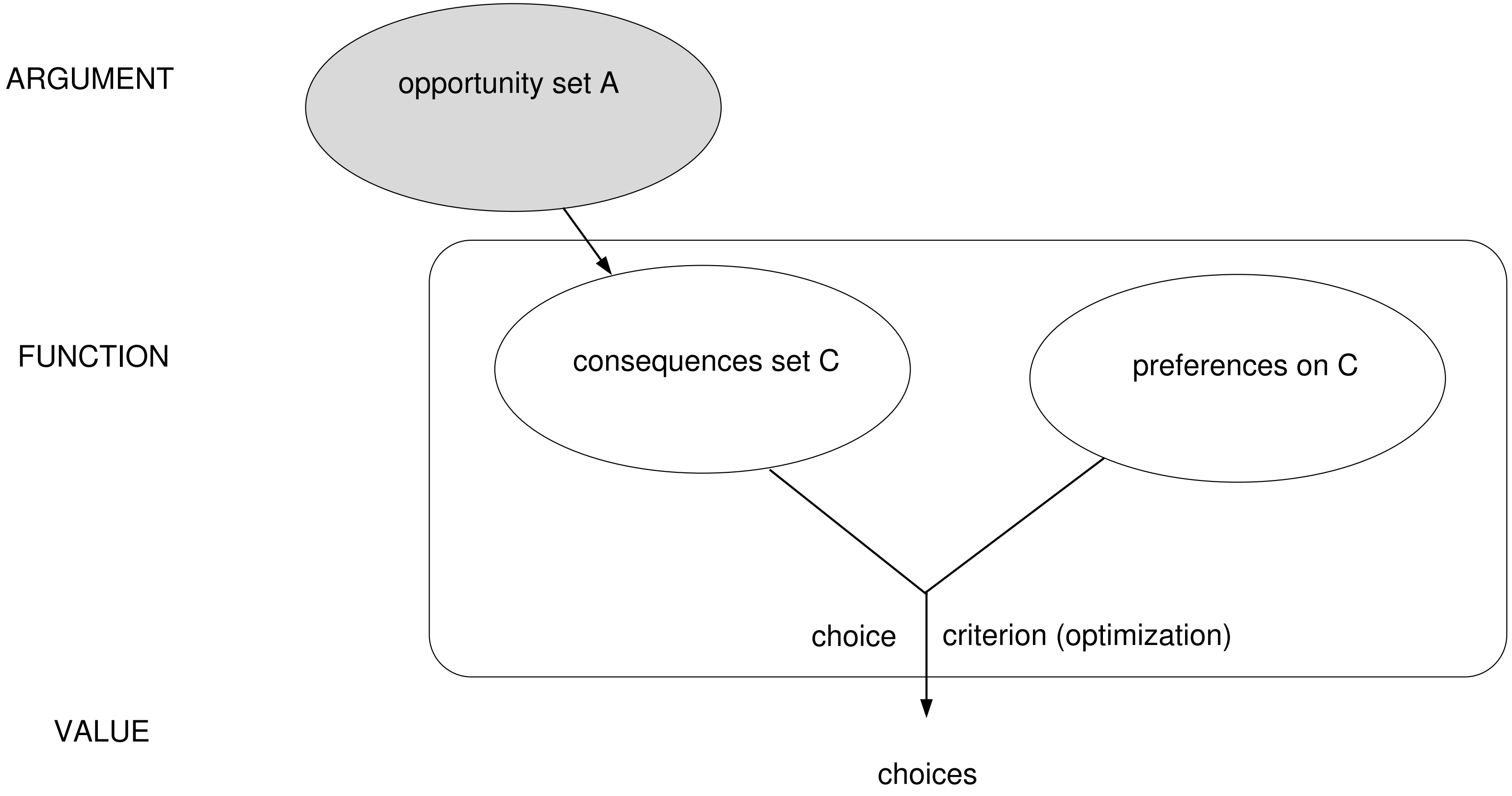
consequences set C

preferences on C

choice criterion (optimization)

choices

sec.1, ECM as a single global function



ARGUMENT

opportunity set A

FUNCTION

consequences set C

preferences on C

choice criterion (optimization)

VALUE

choices

sec.1, computational studies and model's virtues

- Model, modeler and description domain
- Descriptive vs. pragmatic virtues
- Claim :

  (i) contribution *common* to any mathematical model concerns pragmatic virtues

  (ii) contribution *peculiar* to choice models concerns descriptive virtues

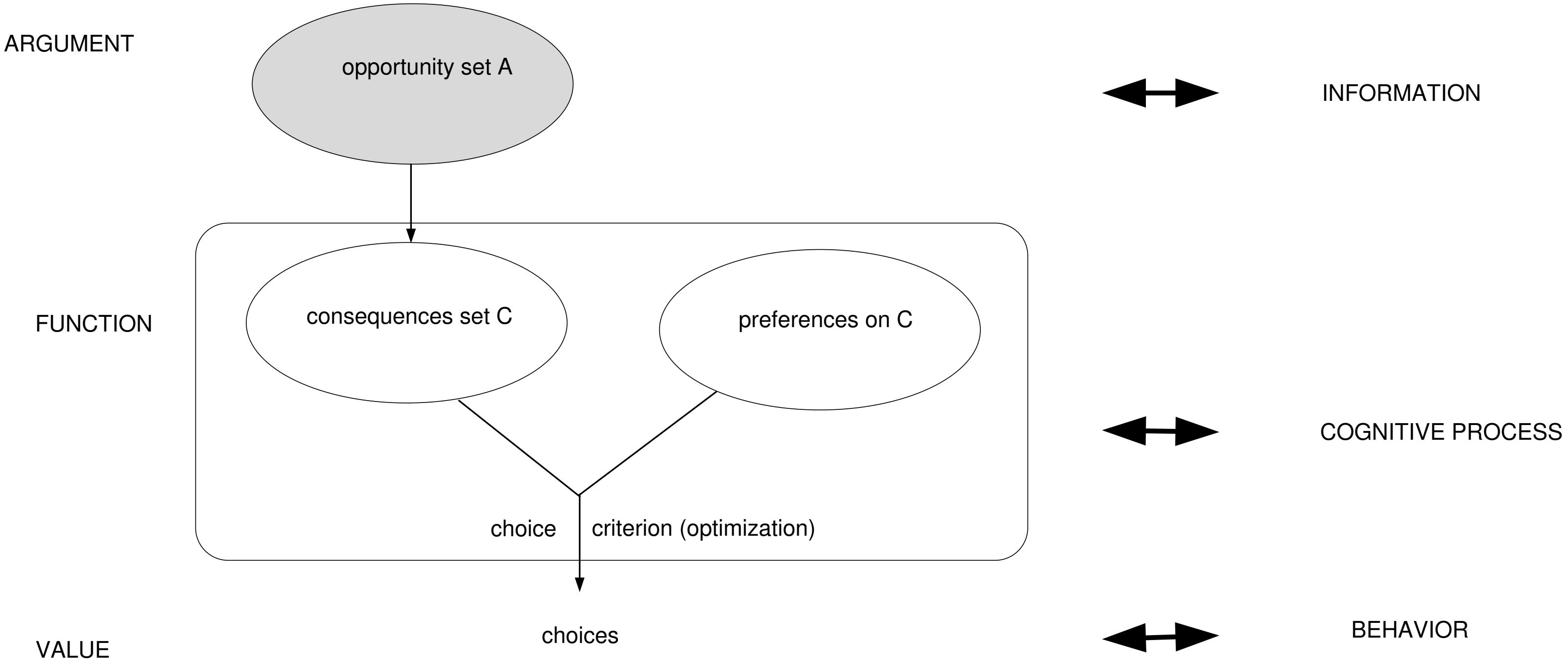
sec.1, computational studies and choice models

- Why a peculiar contribution ?

  (i) *plausible cognitive counterpart condition* : a choice model is descriptively plausible if there is a plausible cognitive counterpart to it.

  (ii) *cognitive relevance of computational studies hypothesis*: computational studies convey information about the plausibility of cognitive competence's ascription.

sec. 1, choice model and cognition



sec.1, computational studies and bounded rationality

- The bounded rationality programm according to Simon : *"The term "bounded rationality" is used to designate rational choice that takes into account the cognitive limitations of the decision-maker - limitations of both knowledge and computational capacity"*.
- The plausible cognitive counterpart condition can be assimilated to the bounded rationality programm.
- The cognitive relevance of computational studies hypothesis can be assimilated to the claim that computational studies are a significant tool to achieve bounded rationality.

sec.1, remark

- *Existence of cognitive counterpart condition* : if a function has no cognitive counterpart, a peculiar contribution of its computational study is not to be expected.
- Example : equilibrium in microeconomics

section 2

Impossibility results

sec. 2, choice functions

- Two main approaches of decision :
  - *preference-based approach* : binary relation $\succeq$ on A ; rational if complete and transitive
  - *choice-based approach* : let A an opportunity set and $\mathbb{F} \subseteq \wp(A)$; a **choice function** for $\mathbb{F}$ is a function $C : \mathbb{F} \longrightarrow \wp(A)$ s.t. $\forall X \in \mathbb{F}$, $C(X) \subseteq X$.
- A choice function is **rational** if there exists a rational preference relation $\succeq$ on A s.t. for all $X \in \mathbb{F}$, $C(X) = \{a : \forall b \in X , a \succeq b\}$. One says that $\succeq$ rationalize $C(.)$.

sec.2, choice function as cognitive function

INFORMATION ↔ X subset of A

COGNITIVE PROCESS ↔ C(.)

BEHAVIOR ↔ C(X)

sec.2, example of impossibility result

- classical framework (consumer theory) : compact and convex set of bundle of n commodities $A \subseteq \mathbb{R}^n_+$
- recursive framework (recursive analysis) :
  - recursive space of opportunities $(R(A), \mathbb{F}_R)$
  - recursive choice function $C : \mathbb{F}_R \to \wp(R(A))$

Theorem (A. Lewis, 1985)
Let A a compact and convex subset of $\mathbb{R}^n_+$. If C is a choice function recursively rational and and non-trivial on $(R(A), \mathbb{F}_R)$, then C is not recursively realizable : for all full domain $\{\mathbb{F}_{Rj}\}$, $graph(C)$ is not recursively solvable.

sec.2, interpretation, claims

Claims :

(i) impossibility results have a true *critical import* for the model's appraisal

(ii) impossibility results invert the onus of the proof

sec.2, claim (i)

Claim :

(i) impossibility results have a true *critical import* for the model's evaluation

- Auxiliary assumption : if a cognitive function is not computable, then it unlikely that an agent can realize it (*unlikehood of non-computability hypothesis*)
- The auxiliary assumption is supported by the computational theory of cognition

sec.2, claim (ii)

Claim :

(ii) impossibility results invert the onus of the proof

- Initial assumption : there is an intuitive evidence for the classical choice model
- The results change the situation ; a model's defender might claim that

a) cognition is not computational $\hookrightarrow$ support ?

b) actual choices need not always to be perfectly rational, suffice to *approximate* the choice model $\hookrightarrow$ support ?

sec.2, computational test

- Computational test of a choice model $\mathcal{M}$
  - select a class of unlikely cognitive function $\mathfrak{F}_U$
  - $\mathcal{M}$ pass the test if its cognitive functions are not in $\mathfrak{F}_U$
  - if $\mathcal{M}$ do not pass the test, at least further reasons for its acceptance than its intuitive evidence

section 3

Computational constraints in repeated games

sec.3, repeated games

Definition
Let $G = ((A_i)_{i \in N}, (u_i)_{i \in N})$ a n-person game ; we define the *t-period repeated game of* G as a the game $G^t = ((S_i)_{i \in N}, (u_i^t)_{i \in N})$ where

(i) S_i is the set of strategies of player i ; a strategy $s_i \in S_i$ is a function $s_i : \bigcup_{k=1}^{t} H^k \longrightarrow A_i$ where $H^k = (A_1 \times ... \times A_n)^{k-1}$ is the set of histories at period k.

(ii) $u_i^t(s) = (\sum_{k=1}^{t} u_i(\omega_k(s)))/t$ where $\omega_k(s)$ is the play at period k induced by the strategy profile $s \in S_1 \times ... \times S_n$.

sec.3, repeated game, example

- G = Prisoner's Dilemma

	C	D
C	$(3,3)$	$(0,4)$
D	$(4,0)$	$(1,1)$

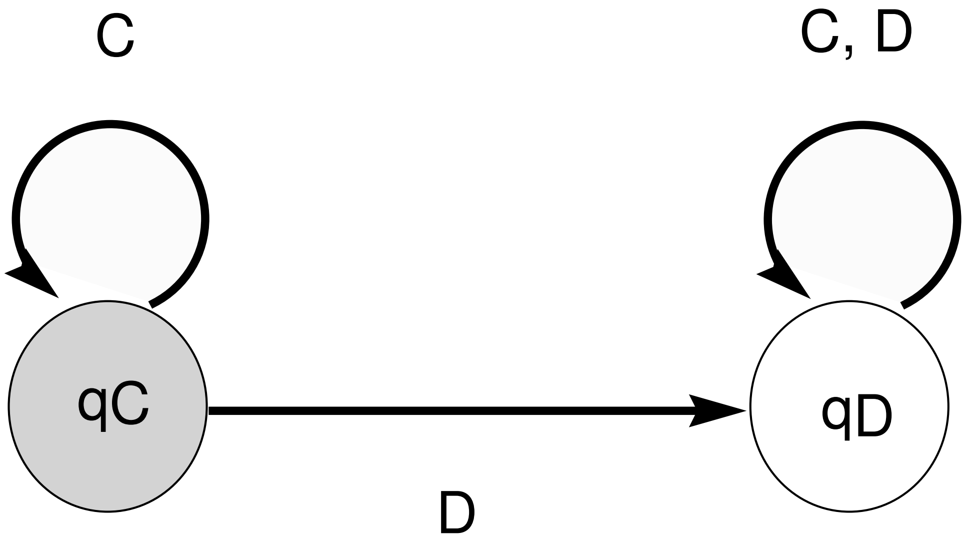
- a *strategy* for a two-period Prisoner's Dilemma specifies
  - what to do in period 1
  - what to do in period 2 given what is played by both players in state 1 (four possibilities)
- "*grim strategy*" : to cooperate but defect forever as soon as the other defects

sec.3, strategies as machines

Definitions

- A finite automaton for player i is a four-tuple $m_i = (Q, q_0, f, \tau)$ where
  (i) Q is the set of automaton's states
  (ii) q_0 is the initial state
  (iii) $f : Q \rightarrow A_i$ is the output function
  (iv) $\tau : Q \times A_{-i} \rightarrow Q$ is the transition function
- A finite automaton m_i induces a strategy s^{m_i} in G^t ; m_i *implements* a strategy $s_i \in S_i$ if $s^{m_i} = s_i$.
- The complexity $comp(s_i)$ of a strategy s_i is the size of the smallest automaton implementing s_i

sec.3, strategies as machines, example



A finite automaton for the grim strategy

sec.3, the amended model

- for each player i, the set of admissible strategies is the set of strategies inferior to a given complexity $r_i(t)$
- the amended model is the same as G^t but with admissible strategies instead of the whole strategy set
- example of impact on model's solutions : cooperation regained in Prisoner's Dilemma

  Theorem

  Let G^t a t-stage Prisoner's Dilemma ; for all $\epsilon > 0$, if $r_1(t)$ or $r_2(t) \leq 2^{c_\epsilon t}$ with $c_\epsilon = \frac{\epsilon}{12(1+\epsilon)}$, then for a t large enough, there exists in $G^t(r_1, r_2)$ a Nash Equilibrium whose payoff is at least $3 - \epsilon$.

sec.3, interpretation, preliminary points

- Admissible vs. available strategies
- What to expect ?

  \- *weak or hypothetical expectation* : amended models introduce a cognitive variable

  \- *strong or categorical expectation* : amended models commit to capture available strategies
- Model vs. theory
- Cognitive improvement

sec.3, interpretation, claims

Claims :

(i) if one looks for a cognitive improvement, then one has to adopt a strong expectation toward amended models

(ii) if one adopts a strong expectation toward amended models, then further requirements, that are usually not satisfied, arise

sec.3, claim (i)

Claim :

(i) if one looks for a cognitive improvement, then one has to adopt a strong expectation toward amended models

sec.3, claim (ii)

Claim :

(ii) if one adopts a strong expectation toward amended models, then further requirements, that are usually not satisfied, arise

- *requirement 1* : empirical support of the computational restriction
- *requirement 2* : systematicity of the computational restrictions
  Problem : to improve one aspect can make worse another one (Papadimitriou 1992)

general conclusion

- relation between appraisal use and improvement use of computational studies
  - not so different that the case-study might suggest
  - the appraisal use check if a given model satisfy a computational test
  - the improvement use build a model that satisfy a computational test
- relation between computationnaly-based and empirically-oriented studies of bounded rationality :
  - computational-based contributions are general, tractable and theoretically fruitful
  - no miracle : the more the restriction is strong, the more it needs to be empirically monitored