On Representation Theorems

What does **Ax** add?

Mikaël Cozic and Brian Hill

University Paris 12 & IHPST / HEC Paris & IHPST

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Representation Theorems

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ARE THESE CLAIMS JUSTIFIED?

Representation theorems

General form

The ingredients:

(I1) **The framework.** A set of objects of choice (alternatives, prospects etc.), $\mathcal{O}$. Often endowed with some "natural" structure.
A preference relation $\preceq$ is assumed on $\mathcal{O}$.

(I2) **The evaluation criterion (EC).** A criterion for determining a preference relation over $\mathcal{O}$ on the basis of functions $f_1, \ldots, f_n$ of aspects of the options. They are taken to be given and considered to be subjective.

(I3) **Axiomatisation (Ax).** A set of axioms on preference relations.

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General form

The Representation Theorem (for a given Evaluation Criterion) states that

$\preceq$ satisfies the axioms **Ax**	$\Leftrightarrow$	there exist (suitably) unique functions $f_1, \ldots, f_n$ of the relevant aspects of $\mathcal{O}$ such that the preference relation determined by $f_1, \ldots, f_n$ according to **EC** coincides with the preference relation $\preceq$

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Some examples: von Neumann Morgenstern

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For example, if the lottery gives 60-40 chances of winning and losing $EUR100$, then the expected utility is $0.6 \times u(100) + 0.4 \times u(-100)$.

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(I3) **Axiomatisation (Ax).** The von Neumann-Morgenstern axioms (**VMM**).
Example: $\preceq$ is transitive and complete.

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The Von Neumann Morgenstern Theorem states that

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For example, if the act yields a $EUR100$ gain if it rains tomorrow and a $EUR100$ loss if not, then the expected utility is $p(Rain) \times u(100) + p(not\ Rain) \times u(-100)$.

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(I3) **Axiomatisation (Ax).** The Savage axioms (**Sav**). Example: Sure-Thing principle (preference between o_1 and o_2 depends only on the states where they differ).

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Savage's Theorem states that

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Foundational ambitions

The starting point:

> *The subject of our inquiry is the logic of partial belief, and I do not think we can carry it far unless we have at least an approximate notion of what partial belief is, and how, if at all, it can be measured. It will not be very enlightening to be told that in such circumstances it would be rational to believe a proposition to the extent of 2/3, unless we know what sort of a belief in it that means. We must therefore try to develop a purely psychological method of measuring belief. (Ramsey, 1931, p166)*

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There are two, strictly speaking different, ambitions involved here:

(i) to elucidate or give the meaning of the concepts involved in **EC**
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It has been claimed that Representation Theorems can provide the meaning or the measurement method, or both.

Representation theorems and the “meaning” of terms ascribing mental attitudes

Let’s accept that:

- preferences are *directly* observable
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Representation theorems and the "meaning" of terms ascribing mental attitudes

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Then the problem of attributing meaning to subjective concepts is analogous to that of attributing meaning to theoretical terms of scientific theories.

Brief résumé of a philosophical approach to defining theoretical terms

The "standard" account (Lewis, 1970) in 3 steps:

1. Start with initial theory featuring theoretical terms $t_1, \ldots, t_n$ as well as other observational terms (and logico-mathematical terms). $T[t_1, \ldots, t_n]$.
2. Remove theoretical terms by replacing them by variables and quantifying. There exists unique $x_1, \ldots, x_n$ such that $T[x_1, \ldots, x_n]$.

3. Define theoretical terms as the unique things which realise the quantification above. t_i is the unique x_i such that there exists unique $x_1, \ldots, x_{i-1}, x_{i+1}, \ldots x_n$ such that $T[x_1, \ldots, x_n]$.

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3. **Def-EC** The agent's utilities "are" the (suitably) unique function u such that there exists a unique p with $o_1 \preceq o_2$ iff $E_p u(o_1) \leqslant E_p u(o_2)$, for all o_1, o_2.

Representation theorems as definitions of theoretical terms

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Hence the question:

- what does the Axiomatisation provided by Representation Theorems add to **Qu-EC**?

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The proposed definition:

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(Triv) **Triviality of the definition**: though theoretical terms are not present on the right hand side of the definition "in letter", they are still present "in spirit" (Hempel, 1958, p216)

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(Hol) **Holism of the definition**: the definition of a theoretical term makes reference to all preferences and to the entirety of the theory.

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Why **Ax**? Some possible replies:

(i) Better understanding of a theory expressed in terms of preferences than in terms of the existence of functions. [(Triv) is undesirable.] **Ax** is in terms of preferences.

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(iv) Want definitions of theoretical terms in order to eliminate all theoretical terms. [(Triv) is undesirable.] **Ax** represents a "real" elimination.

Representation theorems and measurement theory

RTs often presented as results concerning the measurement of subjective concepts (beliefs and utilities).

Indeed, RTs are a well-known class of results in measurement theory.

Aside The measurement question and the meaning question not always clearly distinguished (Cf. Ramsey quote above).

- Operationalist idea: the meaning of a term is given by a method for measuring its value.

Brief résumé of the (formal) problem of measurement

The skeleton of a measurement problem:

- Framework: a domain of objects, endowed with a particular structure.
- Goal: to associate to each object a value (or set of values) in such a way as to “respect” the structure.

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- Central assumption: that a measure exists. Ie. there exists a (suitably) **unique** way of assigning values to objects which "respects" the structure.

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- Eg. there exists a (suitably) **unique** utility function which represents the preference relation.

Representation theorems and measurement theory

The central assumption is none other than the quantification of the Evaluation Criterion **Qu-EC**.

- So what does the axiomatisation add?

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- Similarly, one might argue that **Ax** is easier to evaluate that **Qu-EC**. (See below.)
- A third reply traces a distinction between **measurement in principle** and **measurement in practice**.

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A proof of a representation theorem

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Representation theorems and measurement theory

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Here's what the proof of vNM's representation theorem consists of:

(a) Pick any outcomes A and B such that $B \succ A$

(b) Assign utility values 1 to B and 0 to A.

(c) "Measure" the utility of any C such that $B \succeq C \succeq A$ as follows:

- The utility of C is the real number α such that the subject is indifferent between C and a lottery yielding B with probability α and A with probability $1 - \alpha$.

(d) the rest of the theorem just checks that **VNM** implies that such α always exist and that it is independent of the choice of A and B (up to scaling).

Representation theorems and measurement theory

A proof of a representation theorem

Here's what the proof of vNM's representation theorem consists of:

A "concrete" measurement procedure

Necessary and sufficient conditions for it to always yield consistent results

Representation theorems and measurement theory

An argument for Representation Theorems:

- they provide a concrete measurement procedure for the subjective concepts employed (utility, in the case of **VNM**).
- **Qu-EC** does not.

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- Advantage of the Representation Theorem in itself, and its proof, as much as of the axiomatisation.
- Not a property shared by all axiomatisations.
- Supposes that there is some advantage in having "concrete" measurement procedures.
- The measurement procedure can be thought of as giving an operationalist, explicit, non-holistic account of the meaning of the subjective concept.

Representation theorems as "foundations"

Summing up

If you are moved by	Then you want:
• need for definition in terms of observational terms • guaranty of consistent measurement in principle	**Qu-EC**
• conviction that properties of preferences provide a better understanding than functions	**Ax** as a "conceptual aid"
• molecularism of meaning • need for "concrete" measurement procedure • operationalism	a "constructive" RT and accompanying **Ax**
• eliminativism of theoretical terms	**Ax**

Decision Theory as a Theory of Decision

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- Eg. select a probability function p and utility u and choose the option o with the highest expected utility $E_p u(o)$.

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Hence the content of the theory, as a theory of decisions, is just:

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- What does **Ax** add in each case?

Qu-EC vs. Ax

Two preliminary remarks

Qu-EC vs. **Ax**

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Concerning the last question, there are two manners of proceeding:

- checking with intuition whether the theory seems a reasonable description ("armchair" decision theory)
- compare with field or lab data ("empirical" decision theory)

Testing a theory of decision

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So:

- one way of coming to a decision about the descriptive adequacy of the theory **Qu-EC** is by testing its observable consequences (ie. consequences formulated entirely in terms of preferences).

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- this is what has often been done in practice, eg. experiments refuting EU or alternatives to EU
- the propositions in **Ax** are some but by no means all observable consequences of **Qu-EC**. In fact, consequences can be found and tested without passing through **Ax** (eg. stochastic dominance).

So:

- What does **Ax** add?

Descriptive content: the specificity of **Ax**

What **Ax** has got "over" an arbitrary set of observable consequences of **Qu-EC**:

1. **Ax** axiomatises the set of observable consequences of **Qu-EC**
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Hence,

- From 1.: **Ax** exhausts the observable content of **Qu-EC**.
- From 2.: Establishing the truth of **Ax** is sufficient for establishing the truth of **Qu-EC**.

Thus: **Ax** provides an ideal epistemic position from which to assess **Qu-EC**:

- If one accepts the propositions in **Ax**, one accepts **Qu-EC**.

Using **Ax**

Seems to lead to an "atomistic" method for evaluating **Qu-EC**:

- Check each axiom in **Ax** individually

If you accept each of them (resp. if each is confirmed) then you accept **Qu-EC** (**Qu-EC** is confirmed).

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The "atomistic" access offered by **Ax** may not faithfully reflect the consequences of accepting **Ax**.

Advantages of axiomatisations

Ax brings advantages of axiomatisations which have been recognised in other fields.

In particular, it may play an important "architectonic" role:

(i) in comparing different theories

(ii) in developping new theories

Ax as Architectonics

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The subjective concepts involved in different **EC**'s may be different (or have different "interpretations")

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Added benefit

- Axiomatisations focus attention on properties of preferences which make a difference.
- They add discipline to the debate, by forcing it to take place in a rigorous, observational, common language.

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Theory production

Guiding idea Axiomatisations may play a role in the development of new theories (eg. in the wake of a refutation).

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What motivates the proposal of a new theory (of decision)?

EC-driven An intuition behind a new functional form.

- Eg. Maxmin, α-maxmin.

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Guiding idea Axiomatisations may play a role in the development of new theories (eg. in the wake of a refutation).

What motivates the proposal of a new theory (of decision)?

EC-driven An intuition behind a new functional form.

Property-driven A set of properties of preferences which one would like to have respected.

- Eg. Rank-dependent utilities.

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Although properties of preferences play an important role in the development of new theories, it has not been established that Axiomatisations play a specific supplementary role.

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- but the preceding points related to descriptive assessment seems to be transferable *mutatis mutandis* to normative assessment

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- the RT does not provide a magical shortcut to assess an **EC**
- if one does not accept some joint consequence of **Ax**, then one may question one (or more than one) proposition of **Ax** rather than our normative intuition concerning the joint consequence

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- as concerns the theory *qua* theory of decision, the job in hand – evaluating the theory, arguing for it, comparing theories etc.
- and perhaps: the ambition of the theory – descriptive and normative – or the field in which it is developed – philosophy or economics?

On Representation Theorems

What does **Ax** add?

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University Paris 12 & IHPST / HEC Paris & IHPST

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