Choice and Computation

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introduction (1)

bounded rationality

3 components :

1. factual : agents are facing cognitive limitations
2. critical :given agents' cognitive limitations, classical choice models are inadequate for describing them
3. constructive : one has to build choice models compatible with agents' cognitive limitations

computational studies

- *computability*: is the function f computable ?
- *complexity*: how much resources requires the computation of f ?

introduction (2)

computational studies and bounded rationality

- computational studies claim to be relevant for the understanding of bounded rationality (Kramer 1974, Richter & Wong 1999, Velupillai 2000)
- computational studies are put forward by upholders of bounded rationality (Simon, 1978)
- computational restrictions in the theory of repeated games (Abreu et Rubinstein 1988, Rubinstein 1998, Neyman 1998)
- gaps in methodological analysis (Binmore 1987, Aumann 1997)

aim of the talk

- analysis and assessment of the contribution of computational studies to bounded rationality

introduction (3)

Question 1 : What is the basic connection between computational studies and bounded rationality ?
↪ section 1
Question 2 : How can computational studies help to *appraise* choice models ?
↪ section 2
Question 3 : How can computational studies help to *improve* choice models ?
↪ section 3

Section I

Computational Studies and Bounded Rationality: the Basic Connection

classical choice model under certainty (CMC)

- (M 1) the agent might choose an action in a set A of feasible actions (or opportunities)
- (M 2) agent's preferences on A are represented by a *weak order* $\succeq \subseteq A \times A$ (a complete and transitive binary relation)
- (M 3) the agent chooses a $\succeq$-maximal action (if there is one)

Epistemological Framework

epistemological framework

- model and description domain
  - model : formal structure + generical interpretation
  - description domain : piece of reality whose data are the target of organization, prediction, explanation by means of the model
- descriptive vs. pragmatic virtues :
  - descriptive virtues : model's ability to describe adequately the description domain
  - pragmatic virtues : model's tractability in the study of its description domain

descriptive relevance

computational studies of models

because it is based on a formal structure, every model can be the object of a computational study (*)

physics

- computability : quantum mechanics (Pour-El & Richards, 1989)
- complexity : Ising models in statistical mechanics (Barahona, 1982 , Istrail 2000)

choice

- computability : consumer's choice functions (Lewis, 1985 & 1992), competitive equilibria (Richter & Wong, 1999)
- complexity : subset choice (Fishburn & LaValle, 1996)

descriptive relevance hypothesis

- *common* contribution : information on models' pragmatic virtues
- *specific* contribution : information on models' descriptive virtues = *descriptive relevance hypothesis*

factorization of the descriptive relevance hypothesis

(1) connection choice-cognition

- agents' choices result from a more or less sophisticated cognitive processes ("practical reasoning")
- behavioral adequation vs. cognitive adequation
- correlation between behavioral adequation and cognitive adequation (see experiences based on MouseLab, Costa-Gomes & ali. 2001, Johnson & ali. 2002)
- this view contradicts the "intrumentalist" orthodoxy in the methodology of decision science (see Friedman 1953)

(2) connection computation-cognition

- link between cognitive processes and computational studies
- computational properties as indicators of cognitive abilities

computational studies and bounded rationality

- critical component = classical choice models are cognitively inadequate
- constructive component = one has to build cognitively adequate choice models

cognitive anchoring of computation

- cognition anchors computation in choice models
- when a function has no obvious cognitive interpretation, the descriptive relevance is no longer guaranteed
- example: computational properties of competitive equilibria

Section II

Evaluative use

1. negative results
    1. computability theory (non realizability of choice functions)
    2. complexity theory (**NP**-hardness of subset choice)
2. discussion

target: consumer choice model

choice parameters

- bundles of L goods represented by vectors $x \in \mathbb{R}_+^L$
- prices p, wealth level w
- budget constraint: consumer chooses among $A(p, w) = \{x \in \mathbb{R}_+^L : p.x \leq w\}$

choice functions

- Let A an opportunity set and $\mathbb{F} \subseteq \wp(A)$; a **choice function** for $\mathbb{F}$ is a function $c : \mathbb{F} \longrightarrow \wp(A)$ s.t. $\forall X \in \mathbb{F}$, $c(X) \subseteq X$.
- A choice function is **rational** if there exists a preference relation $\succeq$ on A s.t. for all $X \in \mathbb{F}$, $c(X) = \{a : \forall b \in X$, $a \succeq b\}$. In this case, one says that $\succeq$ rationalize $c(.)$.

framework: recursive analysis

$\mathbb{R}$ (reals)	$\mathbb{R}_c$ (recursive reals)
$A \subseteq \mathbb{R}^n$ (set of feasible actions) $\mathbb{F} = \{X \subseteq A\}$ (subsets of feasible actions)	$R(A) \subseteq M(\mathbb{R}^n)$ (recursive set of feasible actions) $\mathbb{F}_R = \{X : X \subseteq R(A) \wedge \text{X recursive}\}$ (recursive subsets of feasible actions)
$c : \mathbb{F} \longrightarrow \wp(A)$	$c : \mathbb{F}_R \longrightarrow \wp(R(A))$

Définition

A choice function c on $(R(A), \mathbb{F}_R)$ *is* **recursively rationalizable** *if there exists*

(i) *a relation* $\succeq: R(A) \times R(A) \rightarrow \{1, 0\}$
(ii) *a recursive partial function* $f : R(A) \rightarrow \mathbb{N}$ *s.t.* $\forall a, b \in R(A)[(a \succeq b) = 1 \rightarrow f(a) \geq f(b)]$ *and* $\forall X \in \mathbb{F}_R$, $c(X) = [a : \forall b \in X(f(a) \geq f(b))]$.

Définition

Given a domain $\{\mathbb{F}_{Rj}\}_{j\in\mathbb{N}} \subseteq \mathbb{F}_R$ *et un co-domaine* $\{c(\mathbb{F}_{Rj})\}_{j\in\mathbb{N}}$*, the* **graph** *of c is the set of pairs* $(\mathbb{F}_{Rj}, c(\mathbb{F}_{Rj}))$*. The graph of c has* **full domain** *if for a* $K \in \mathbb{N}$ *and for each pair* $i \neq j > K$, $\mathbb{F}_{Ri} \triangle \mathbb{F}_{Rj} \neq \emptyset$.

Définition

A recursively rationalizable choice function on $(R(A), \mathbb{F}_R)$ *is* **recursively realizable** *iff for every full domain* $\{\mathbb{F}_{R_j(j\in\mathbb{N})} \subseteq \mathbb{F}_R\}$*, the graph of C is a recursive set of the space* $\wp(M(\mathbb{R}^n)) \times \wp(M(\mathbb{R}^n))$.

impossibility result

theorem (Lewis, 1985)

Let c a non-trivial recursively rationalizable choice function on $(R(A), \mathbb{F}_R)$, then c is not recursively realizable and $\{\mathbb{F}_{Rj}\}$ is a full domain. The graph of c is not recursively realizable.

complexity theory

motivations

- computability by TM *vs.* computability in practice or feasible computability
- complexity theory develops notions that are supposed to be closer to computability in practice
- measure of spatial and temporal resources
- **P** vs. **NP**

negative result

target: model of subset choice

- finite set of objects O; each object $x \in O$ has a price $p(x)$ and each subset $X \subseteq O$ has a price $p(X) = \sum_{x \in X} p(x)$
- linear utility function $u(X) = \sum_{a \in X} u(a)$
- solution $sol(O, p, w, u) = \arg\max_{X \subseteq O : p(X) \leq w} u(X)$

proposition (Fishburn & LaValle 1996)

sol is **NP***-hard.*

discussion (1)

claim

Negative results have a true critical import for the target choice models from the descriptive point of view

computability case

non recursivity

$\Downarrow$

computational impossibility

$\Downarrow$

cognitive unlikelihood

$\Downarrow$

behavioral unlikelihood

computational test

Computational test of M :

Step 1 : one picks a class $\mathfrak{F}_I$ of "cognitively likely" functions on the basis of computational criteria

Step 2 : M is subjected to a *computational test with respect to* $\mathfrak{F}_I$: M passes the test if the functions associated to M which have cognitive interpretations are in $\mathfrak{F}_I$.

discussion (3)

what might one infer from a failure to pass the test ?

- strong reaction: reject a model M that do not pass the test with respect to a reasonable class of "cognitively likely" functions
- failure to pass the test is not sufficient to reject the model
- for instance, approximation is not excluded
- failure reverses the onus of the proof

the "Easy Problems"

"Easy Problems"

- step 1 ⇒ *psychological questions:* what is the precise cognitive adequacy of such and such computational criterion ? (cf. van Benthem 2006, computational complexity vs. cognitive difficulty)
- step 2 ⇒ *mathematical questions:* does a given choice model M pass the test for a given computational criterion ?

Section III
Constructive use

finitely repeated games

classical setting

- basic game $G = ((A_i)_{i \in N}, (u_i)_{i \in N})$
- at each stage of the t-repeated game G^t, players play the game G
- at stage $k \leq t$, agents will choose their actions depending on what happened in preceding stages *i.e.* depending on the *history* of the play
- agents' opportunities in G^t are *strategies* *i.e.* functions that associates (basic) actions to every possible history
- in G^t, agents' utilites are the average of the payoffs they receive at each stage of the play

computational restrictions on strategies

computational restrictions

- combinatorial explosion of the set of available strategies
- some strategies are (intuitively) simple, some may be extremely sophisticated
- basic idea: to cancel the hardest strategies from the opportunity set
- assumption: the (intuitive) complexity of a strategy can be measured by the size of the smallest finite automaton that can implement it
- theoretical investigation: how the outcomes of the game change when one fixes upper bound on the measure of the (intuitive) complexity of strategies

boomerang effect

- the computational amendment concerns choice parameters (more precisely opportunities) and not model's solution
- agents are still supposed to conform to Nash equilibria and to play their best strategies given the strategies played by other agents
- the amendment is therefore very *partial*, it doesn't improve the crucial maximizing assumption of classical model
- partiality might make things worse; as a matter of fact, Papadimitriou (1992) has shown that the problem of finding a best response to a given strategy is tractable without restrictions but intractable with restrictions

the Hard Problem

the Hard Problem

- let's consider a classical choice model M with (maximizing) solution concept sol_M
- let's suppose that the class of "cognitively likely" functions is $\mathbb{F}_l$ and that $sol_M \notin \mathbb{F}_l$
- which substitute for sol_M ?

conclusion

main points

1. defense of the use of computational studies for bounded rationality that is grounded on cognition
2. distinction between Easy Problems and Hard Problem

references (1)

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